A BRIEF HISTORY OF THE YIELD STRESS

HOWARD A. BARNES

Unilever Research, Port Sunlight, L63 3JW, U.K.
e-mail: howard.barnes@unilever.com
Fax: x44.151.6411829

Received: 20.4.1999, Final version: 1.6.1999

ABSTRACT
This short article is a summary of a recently published, lengthy review [1] in which the author challenges the often-accepted view that materials have yield stresses, below which no flow takes place. However, following the introduction of the new generation of controlled-stress rheometers it is shown that when careful measurements are made below the supposed 'yield stress', flow does actually take place. The argument for the non-existence of the yield stress as a physical entity now seems insuperable. However, in spite of this, it is nevertheless accepted that an apparent yield stress is a useful mathematical abbreviated description of limited data over a given range of flow conditions. This yield stress parameter can be used effectively for predicting flow, but only within the region of the original measurement that furnished the yield stress.

KURZFASSUNG
Dieser kurze Artikel ist eine Zusammenfassung eines veröffentlichten, längeren Übersichtsbeitrages, in dem der Autor die häufig vertretene Ansicht bestreitet, dass Stoffe eine Fließgrenze besitzen [1]. Gemäß der Einführung einer neuen Generation von spannungsgeregelten Rheometern wurde gezeigt, dass, wenn Messungen unterhalb der Fließgrenze sorgfältig genug ausgeführt werden, tatsächlich Fließen feststellbar ist. Das Argument für das Nicht-Existieren der Fließgrenze als eine physikalische Größe scheint unumstößlich zu sein. Dennoch ist es ein akzeptabeles Argument, dass die scheinbare Fließgrenze eine nützliche, mathematisch abgekürzte Beschreibung von begrenzten Messdaten über dem vorliegenden Strömungsfall gestattet. Der Parameter Fließgrenze kann sehr effektiv genutzt werden, um Fließen vorherzusagen, jedoch nur für die Messung, die der Bestimmung der Fließgrenze zugrundelag.

RÉSUMÉ
Ce court article est un résumé d'une longue revue récemment publiée, dans laquelle l'auteur défie l'idée souvent acceptée que les matériaux possèdent une contrainte seuil en dessous de laquelle aucun écoulement n'existe. En fait, grâce à l'introduction d'une nouvelle génération de rhéomètres à imposition de contraintes, il est montré que, lorsque des mesures sont faites à des contraintes inférieures à la contrainte seuil supposée, un écoulement existe en réalité. L'évidence de la non existance d'une contrainte seuil en tant qu'entité physique semble maintenant inattaquable. Malgré cela, il est néanmoins accepté qu'une contrainte seuil apparente est une description mathématique raccourcie mais utile de données expérimentales limitées à un régime d'écoulement donné. Ce paramètre de contrainte seuil peut effectivement être utilisé pour prédire l'écoulement, mais seulement dans le régime d'écoulement où la première mesure, conduisant à la définition de la contrainte seuil, a été faite.

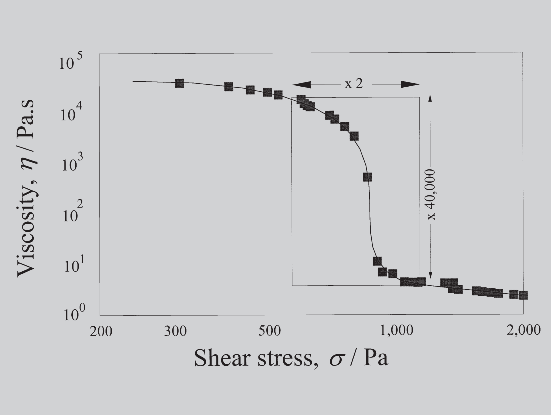
KEY WORDS: Yield stress, Controlled-stress rheometry, History

1 INTRODUCTION

Thomas Jefferson once said that 'difference of opinion leads to enquiry, and enquiry to truth', this has certainly been true with regard to the yield stress of structured liquids over the last century. I have written my version of this story in an extensive review recently published in the Journal of Non-Newtonain Fluid Mechanics [1]. Here is a brief version of that story, but only taking a tenth of the space of the original.

Special attention has been paid over the years to very shear-thinning liquids that *appear* to have a yield stress. It was usually thought that at stresses below the yield stress no flow takes place, and only elastic behaviour was seen. This is in fact only half the story, since indeed although there are materials which *appear* to show this kind of behaviour, in reality there is as much happening in terms of flow below as above the 'yield stress'. Figs. 1 - 9 show a number of examples of such

This is an extract of the complete reprint-pdf, available at the Applied Rheology website
http://www.appliedrheology.org



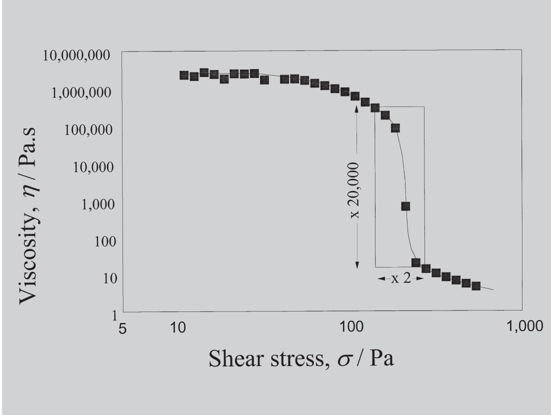


Figure 7 (left): Flow curve of a flocculated ink.

Figure 8 (right): Flow curve of a toothpaste.

via the couple applied to a typical geometry is relatively easy compared to generating the equivalent range of shear rates that would need to be applied. *Measuring* a wide range of shear rates is much easier than *generating* them, and this measurement is done very accurately these days using optical discs.

5 THE VANE GEOMETRY

Last of all in our consideration of appropriate equipment to measure materials that appear to have a yield stress, we must consider the steps necessary to avoid wall-depletion or slip artefacts that very often arise in low shear rate measurement of the kind of highly structured liquids that appear to have yield stresses, see figure 10. Apart from roughening the walls of existing geometry members, the most widely used geometry suitable for this purpose of eliminating wall effects is the vane geometry (where the vane is used instead of an inner cylinder of a viscometer / rheometer), with its diversity as to vane number and aspect ratio. It offers the possibility of inserting the thin vanes into an existing rested or stored sample with the minimum of disturbance.

Theoretical analyses have shown that the vane geometry acts as a circumscribing cylinder defined by the tips of the vane blades, with the material inside the virtual cylinder essentially acting as a solid body, and the material outside being sheared in the normal way. This ensures that slip is completely overcome at the rotating member.

Barnes [3] went further than simply using a vane when he introduced a close-fitting wire-mesh cylinder inside the outer containing cylinder to prevent slip there also, which can often occur, but is usually ignored. (Of course, use of a vane geometry at very high rotation rates is precluded due to secondary flow developing behind the vanes.)

6 SOME FLOW EQUATIONS WITH YIELD STRESSES

Many of the liquids whose flow curves we have just examined will obviously appear to have a yield stress if scrutinised only over the shear-rate/shear-stress region just above the large increase in viscosity. Much useful progress has been made for these kinds of liquids by using simple, yield-stress-containing equations that fit steady-state, shear-stress/shear-rate data quite well over a *limited* range of shear rates. Then, if these equations are used to predict flow in any other situation, where *the same general range of shear rates applies*, this is completely acceptable and un-controversial, and can indeed be very effective.

The most popular equations that have been used to describe liquids with yield stresses are the Bingham, Casson and Herschel-Bulkley (sometimes called the generalised Bingham) models, *i.e.*

$$\sigma = \sigma_y + \eta_p \dot{\gamma} \qquad \text{Bingham}$$

$$\sqrt{\sigma} = \sqrt{\sigma_y} + \sqrt{\eta_p \dot{\gamma}} \qquad \text{Casson}$$

$$\sigma = \sigma_y + k\dot{\gamma}^n \qquad \text{Herschel - Bulkley}$$

Barnes et al [4] showed that the Bingham model is the most non-Newtonian example of the Sisko model, which is itself a simplification of the Cross (or Carreau model) under the appropriate conditions.

A simple but versatile model that can cope with a yield stress, yet retain the proper extreme of a finite zero and infinite shear-rate viscosity is the Cross model with the exponent set to unity, so

$$\frac{\eta - \eta_\infty}{\eta_o - \eta_\infty} = \frac{1}{1 + k\dot{\gamma}}$$

This is an extract of the complete reprint-pdf, available at the Applied Rheology website
http://www.appliedrheology.org

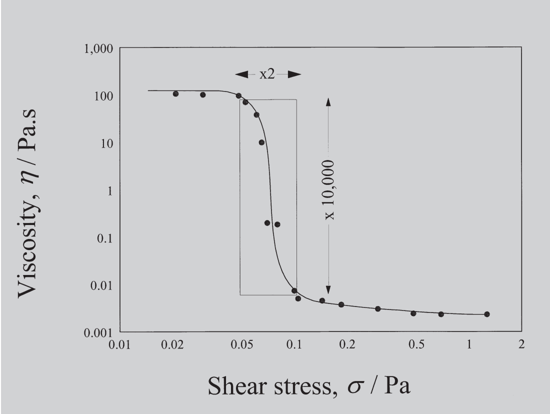


Figure 9 (left): Flow curve of saliva at room temperature.

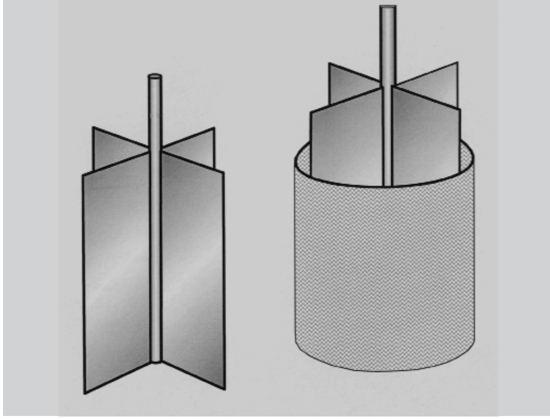


Figure 10 (right): The vane and the vane-and-basket geometries.

the behaviour of which is shown in Figs. 11 and 12, plotted either linearly as stress against shear rate or logarithmically in terms of stress versus shear rate or viscosity versus stress. This behaviour is then very similar to Figs. 1 to 9, where we note the large drop in viscosity for only a moderate increase in stress for all the results shown. Above the 'yield stress' where $\eta << \eta_o$, and $k\dot{\gamma} >> 1$ and it is easy to show that the equation simplifies to

$$\sigma = \frac{\eta_o}{k} + \eta_\infty \dot{\gamma}$$

which is the same as the Bingham equation, with $\sigma_o = \eta_o / k$ and $\eta_p = \eta_o$.

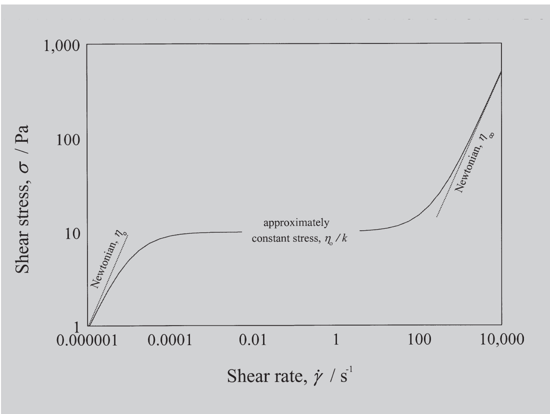


Figure 11: The Cross model with m=1 and typical values of the parameters, showing the apparent yield stress region and upper and lower Newtonian regions.

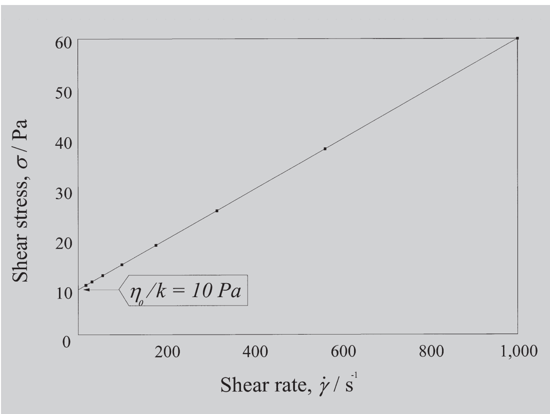


Figure 12: The Cross model with m=1, plotted linearly to show Bingham-type behaviour.

7 CONCLUSIONS

The arguments given for the non-existence of a yield stresses are not new. With regard to liquid-like materials, Reiner had said long ago that 'there is no yield point' [5], and then Coleman, Markovitz and Noll [6] stated in 1966 that 'The concept of a plastic material has been greatly overworked. Almost without exception the yield values reported have been obtained by extrapolation of limited data. When careful measurements are made below the 'yield stress', it is found that flow actually does take place'. To these statements we have added new evidence from the new generation of controlled-stress rheometers. These argument for the non-existence of a yield stress now seems insuperable, however we still anticipate new evidence! In spite of the non-existence of a yield stress as a physical entity, it is nevertheless a useful mathematical abbreviated description of limited data over a given range of flow conditions. If you have found this article interesting, please read the complete account of the yield stress story in my review [1].

REFERENCES

[1] Barnes, H. A: *Yield stress – a review, or παντα ρει – everything flows?*, J. Non-Newtonian Fluid Mech. 81 (1999) 133.

[2] Sherman, P.: *Industrial Rheology*, Academic press, London, 1970.

[3] Barnes, H. A: J. Non-Newtonian Fluid Mech. 56 (1995) 221.

[4] Barnes, H. A., Hutton, J. F. and Walters, K.: *An Introduction to Rheology*, Elsevier, Amsterdam 1989.

[5] Reiner, M.: *Ten Lectures on Theoretical Rheology*, Rubin Mass, Jerusalem, 1943.

[6] Coleman, B. D., Markovitz, H. and Noll, W.: *Viscometric Flows of Non-Newtonian Fluids - Theory and Experiment*, Springer, Berlin, 1966.

This is an extract of the complete reprint-pdf, available at the Applied Rheology website
http://www.appliedrheology.org