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Simple Scalar Model and Analysis for Large Amplitude Oscillatory Shear

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Abstract:
This work presents a simple, scalar model for predicting a nonlinear shear stress response of a viscoelastic fluid in Large Amplitude Oscillatory Shear (LAOS) experiments. The model is constructed by replacing the viscosity in the well-known Maxwell model by a shear rate dependent viscosity function. By assuming the empirical Cox-Merz rule to be valid, this shear rate dependent viscosity function is specified based on the Maxwell expression for the complex viscosity. We thus construct a particular case of the White-Metzner constitutive equation. Numerical solutions as well as an asymptotic analytical solution of the model are presented. The results, analyzed for higher harmonic content by Fourier transform, are compared to experimental data of a viscoelastic solution of wormlike micelles based on cetyltrimethylammonium bromide. Good agreement is found for low frequencies, where viscous properties dominate.

Key words:
Large Amplitude Oscillatory Shear (LAOS), constitutive modeling, Cox-Merz rule, Maxwell model, non-linear rheology, wormlike micelles

1 INTRODUCTION

Nonlinear viscoelasticity plays a crucial role for the mechanical behavior of complex fluids (e.g. polymer melts, polymer solutions, and dispersions) under many processing and application conditions. The use of Large Amplitude Oscillatory Shear (LAOS) experiments, where a sample is subjected to a sinusoidal shear deformation $\gamma(t) = \gamma_0 \sin(\omega t)$ has become a common technique to probe nonlinear viscoelasticity of materials [1–6]. Its main advantage is the possibility to investigate the effect of both characteristic dynamic variables, the Deborah number *De* and the Weissenberg number *Wi* using the same test with the most common rheological equipment, a rotational rheometer. The Deborah number $De = \lambda/\tau_o$ is defined as the ratio of a characteristic relaxation time of a material λ and a characteristic time of observation τ_o, which for oscillatory flow is the inverse of the angular frequency $\tau_o = 1/\omega$. This *De* measures to which degree elastic effects influence the overall mechanical response. The Weissenberg number $Wi = \lambda/\tau_d$ is the ratio of λ and a characteristic time of the deformation τ_d. For steady shear τ_d is the inverse of the shear rate $\tau_d = 1/\dot{\gamma}$ whereas for oscillatory shear $\tau_d = 1/\dot{\gamma}_o = 1/(\omega\gamma_o)$ [7], where $\dot{\gamma}_o$ denotes the shear rate amplitude. The Weissenberg number can be interpreted as a dimensionless shear rate, indicating the influence of nonlinear behavior. Further advantages of using LAOS to probe nonlinear viscoelasticity include the omission of sudden signal jumps in the strain input, as in step experiments, and the ability to probe large strain rates without edge failure [8].

Recent efforts in constitutive modeling of LAOS behavior have led to approximate solutions that provide material functions for a couple of nonlinear models. Whereas some of these are truncated expansions in the shear rate amplitude (corotational Maxwell [9]), or in the shear strain amplitude (Giesekus [10] and Pom-Pom [11]). Others are asymptotic solutions, such as the molecular stress function model [12, 13] and a thixotropic

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Equation A.26 shows that the *De* dependence of Q_o is quadratic for small *De*, whereas the asymptotic limit for large *De* is a linear function of *De*:

$$\lim_{De\to o} Q_o = \frac{(1-c)}{24} De \tag{A.27}$$

Combining Equations A.26 and A.27 results in Equation A.28, which captures both limiting behaviors.

$$Q_{o,a} = \frac{(1-c)}{8} \frac{De^2}{(1+3De)} \tag{A.28}$$

The approximate function Q_o,a has then the form of Equation 16 but is inexact in comparison to Equation A.26 around *De* = 1.

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