Applied Rheology | DOI: 10.3933/APPLRHEOL-26-24246 | WWW.APPLIEDRHEOLOGY.ORG

Resonances in Oscillatory Rheometry

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Received: 30.6.2015, Final version: 16.2.2016

Abstract:
Resonance phenomena are discussed in detail. The influence of significant parameters as the moment of inertia and the measuring constants are enlightened and verified with measurements. It is shown that resonance frequencies weekly depend upon the moment of inertia and strongly on the geometrical coefficient of the measuring system. Both parameters form the configuration constant. If a measuring system is replaced, the moment of inertia changes little but the configuration constant changes more. Thus resonance frequencies can be shifted some decades. The comparison between the developed formalism and measurements gives good results for different rheological measuring modes. Even at pronounced resonances measurements provide proper results. The formalism can be used for the simulation of measuring values. However, deformation oscillations along the rotating axis generate resonances of higher order at higher frequencies. These phenomena contribute systematically errors and should be avoided.

Key words:
Oscillation rheometry, resonance, moment of inertia, measuring modes, frequency sweeps, controller

1 INTRODUCTION

It is well known that oscillating systems show resonance phenomena potentially leading to resonance destruction. In Figure 1 a typical resonance behaviour is shown. The applied torque M_o in the frequency regime is constant. The sample torque amplitudes M_{So} already pass a maximum at frequency ω_s and the angle amplitudes also pass a maximum at frequency $\omega_r < \omega_s$. As this fact cannot be neglected for rheological oscillatory testing some questions arise: What is resonance and how it impacts rheological measurements? Can resonance be predicted? Does resonance frequency depend upon the measuring mode? How can resonance frequency be shifted? Must resonance be avoided and if yes, how can this be done? The theoretical discussion delivers methods to understand the frequency dependences of all physical as well as rheological variables. If the constitutive equation of the sample is known then the applied exciting torque, the sample torque, the angle response and all corresponding rheological variables can be calculated. The knowledge of the mechanical variables is necessary and sufficient to identify a resonance. The formalism is derived from the torque balance equation. First it is discussed for constant applied torque and constant sample parameters. Then the formalism is generalized for any constitutive equation and any measuring mode like controlled shear stress (CSS = CS), controlled shear rate (CSR = CR), controlled shear deformation (CSD = CD), direct strain oscillation (DSO), or TruStrain. Finally the formalism is compared with CSS measurements on an elastomer and CSS and TruStrain measurements on PDMS. Thus the formalism allows simulation of measurements for any samples. This contribution discusses in details the influencing factors on resonances in different measuring modes and enlightens the role of moment of inertia and other important parameters in oscillatory measurements.

2 RHEOMETRIC MEASURING MODES

Rheometric measurements are divided into rotational and oscillatory measurements. In rotational rheometry two main modes are distinguished: The controlled shear stress (CSS = CS) and the controlled shear rate (CSR = CR) mode. In the first mode an applied torque is pre-set and the resulting angular velocity is the response. In the latter case the angular velocity is pre-set and the torque has to be detected, which realises the pre-set angular velocity. Provided we have a torque driven motor for instance a drag cup design or a synchronous motor or an electronically commutated drive. The pre-setting of an angular velocity works as follows: A

LAOS regime this no longer is valid [6]. Instead of real numbers at different frequencies one can use a real function of frequency. This can be utilized to calculate the frequency dependent behavior of all torques and of the angle response in CSS mode and CSD and DSO mode as outlined in the next chapter.

$$\begin{aligned} M_o &\rightarrow \phi_o(\omega^2) \\ &\rightarrow M_{So}(\omega^2) \\ a\cdot M_o &\rightarrow a\cdot\phi_o(\omega_1^2) \\ &\rightarrow a\cdot M_{So}(\omega_1^2) \\ b\cdot M_o &\rightarrow b\cdot\phi_o(\omega_2^2) \\ &\rightarrow b\cdot M_{So}(\omega_2^2) \end{aligned} \tag{A.25}$$

This enables the simulation of measurements in different controlling modes including all mechanical variables.

A.4.1 Resonance at constant sample torque amplitude: another CSS mode

To simulate CSS mode with constant pre-set $M_{S,preset}$ the applied torque amplitude M_O, the angle amplitude response $\phi_O(\omega^2)$, and the sample torque amplitudes $M_{SO}(\omega^2)$ are divided by the frequency dependent sample amplitude function $M_{SO}(\omega^2)$ and multiplied with the constant pre-set amplitude $M_{S,preset}$. Equation A.26 summarizes all conversions and Equation A.27 presents the converted function for the applied torque amplitudes and Equation A.28 the corresponding angle amplitude responses.

$$\begin{aligned} M_o(M_{S,preset}) = M_o\frac{M_{S,preset}}{M_{So}(\omega^2)} &\rightarrow \phi_o(\omega^2)\frac{M_{S,preset}}{M_{So}(\omega^2)} \\ &\rightarrow M_{So}(\omega^2)\frac{M_{S,preset}}{M_{So}(\omega^2)} = M_{S,preset} \end{aligned} \tag{A.26}$$

$$\begin{aligned} M_o\frac{M_{S,preset}}{M_{So}(\omega^2)} &= M_{S,preset}\frac{\left((\omega_o^2-\omega^2)^2+(2\chi\omega)^2\right)^{\frac{1}{2}}}{\left(\omega_o^4+(2\chi\omega)^2\right)^{\frac{1}{2}}} \\ &= M_{S,preset}\frac{f_2(\omega^2)^{\frac{1}{2}}}{f_1(\omega^2)^{\frac{1}{2}}} \end{aligned} \tag{A.27}$$

$$\begin{aligned} \phi_o\frac{M_{S,preset}}{M_{So}(\omega^2)} &= M_{S,preset}\frac{\phi_o}{\left(\omega_o^4+(2\chi\omega)^2\right)^{\frac{1}{2}}J\phi_o} \\ &= \frac{M_{S,preset}}{J}\frac{1}{f_1(\omega^2)^{\frac{1}{2}}} \end{aligned} \tag{A.28}$$

The applied exciting torque (Equation A.26) becomes frequency dependent as can be seen in Figure 2. The constant M_O was divided by the function $M_{SO}(\omega^2)$ which has a maximum behavior at ω_S. That must produce a minimum behavior in the corresponding exciting torque at ω_S (Equation A.27). The first derivative of the exciting torque at constant sample torque amplitude (Equation A.29) delivers the negative difference of Equation A.22 and is set to zero. Of course, this gives the same resonance frequency ω_S as shown in Equation A.24.

$$\omega_S = \frac{\omega_o^2}{2\chi}\sqrt{\sqrt{1+\frac{2(2\chi)^2}{\omega_o^2}}-1} = \frac{G'}{\eta'}\sqrt{\sqrt{1+\frac{2\eta'^2}{CG'}}-1} \tag{A.29}$$

The angle response at constant sample torque amplitude (Equation A.28) and the first derivative (Equation A.30) show monotonic decreasing behavior with angular frequency which also can be seen in Figure 2. Hence pre-setting M_{SO} avoids large angle amplitudes, the sample is possibly kept in its linear viscoelastic range.

$$\frac{d}{d(\omega^2)}\left(\frac{\phi_o}{M_{So}(\omega^2)}\right) = -\frac{M_{S,preset}}{J}\frac{2\chi^2}{f_1(\omega^2)^{\frac{3}{2}}} = 0 \tag{A.30}$$

$$\omega^2 \rightarrow \infty \tag{A.31}$$

One can calculate a resonance reduction ρ_O and a half-power bandwidth $\Delta\omega$ for $M_O(\omega^2)$ which in the case of small damping lies near the values calculated in Equations A.18 and A.20. If in a frequency sweep with constant applied torque amplitude a resonance occurs the angle responses pass a maximum which can deform the sample outside the linear viscoelastic region. If in a frequency sweep at constant sample torque amplitude a resonance occurs the angle responses are monotonic decreasing with increasing frequency. The applied torque amplitudes then pass a minimum. The occurrence of an angle maximum can be avoided by pre-setting a constant sample torque amplitude.

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A.4.2 Resonance at constant angle amplitude: CSD or DSO mode

To simulate CSD or DSO respective TruStrain mode with constant pre-set $\phi_{o,preset}$ the applied torque amplitude M_o, the angle response ϕ_o, and the sample torque amplitudes M_{So} are divided by the frequency dependent angle amplitude function $\phi_o(\omega^2)$ and multiplied with the constant pre-set amplitude $\phi_{o,preset}$. Equation A.32 summarizes all conversions. Equation A.33 presents the converted function for the applied torque amplitudes and Equation A.34 for the corresponding sample torque amplitudes.

$$M_o \frac{\phi_{o,preset}}{\phi_o(\omega^2)} \quad \rightarrow \quad \phi_o(\omega^2)\frac{\phi_{o,preset}}{\phi_o(\omega^2)} = \phi_{o,preset}$$
$$\rightarrow \quad M_{So}(\omega^2)\frac{\phi_{o,preset}}{\phi_o(\omega^2)} \tag{A.32}$$

$$M_o \frac{\phi_{o,preset}}{\phi_o(\omega^2)} \quad = \quad \phi_{o,preset}\, J\, f_2(\omega^2)^{\frac{1}{2}} \tag{A.33}$$

$$M_{So}(\omega^2)\frac{\phi_{o,preset}}{\phi_o(\omega^2)} \quad = \quad \phi_{o,preset}\, J\, f_1(\omega^2)^{\frac{1}{2}} \tag{A.34}$$

The applied exciting torque amplitudes become frequency dependent. The constant M_o was divided by the function $\phi_o(\omega^2)$ which has a maximum behavior at ω_r. That must produce a minimum behavior in the corresponding applied torque at ω_r as calculated in Equations A.35 and A.36 and shown in Figure 3.

$$\frac{d}{d(\omega^2)}\left(\frac{M_o}{\phi_o(\omega^2)}\right) = J\frac{\omega^2 - \omega_o^2 + 2\chi^2}{f_2(\omega^2)^{\frac{1}{2}}} = 0 \tag{A.35}$$

$$\omega^2 = \omega_o^2 - 2\chi^2 \equiv \omega_r^2 \tag{A.36}$$

The sample torque amplitudes at constant pre-set angle amplitude (Equation A.34) and the first derivative (Equation A.37) show monotonic behavior with angular frequency.

$$\frac{d}{d(\omega^2)}\left(\frac{M_{So}}{\phi_o(\omega^2)}\right) = J\frac{2\chi^2}{f_1(\omega^2)^{\frac{1}{2}}} \tag{A.37}$$

The resonance reduction ρ_o and a half-power bandwidth $\Delta\omega$ for $M_o(\omega^2)$ can again be calculated according to Equations A.18 and A.20. If in a frequency sweep at constant angle amplitude a resonance occurs then the applied torque amplitudes pass a minimum. The occurrence of an angle maximum is already avoided by presetting a constant angle amplitude.

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