

NUMERICAL COMPUTATION OF EXTRUSION AND DRAW-EXTRUSION CABLE-COATING FLOWS WITH POLYMER MELTS

A. AL-MUSLIMAWI^{1,2}, H.R. TAMADDON-JAHROMI³ AND M.F. WEBSTER^{3*}

¹ Swansea University, College of Science, Mathematics, Singleton Park, Swansea SA2 8PP, United Kingdom

² Basra University, College of Science, Mathematics, Baghdad Street, AL-Basra, 61004, Iraq

³ Swansea University, Institute of Non-Newtonian Fluid Mechanics, Singleton Park, Swansea SA2 8PP, United Kingdom

*Corresponding author: m.f.webster@swansea.ac.uk

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ABSTRACT:

This paper is concerned with the numerical solution of polymer melt flows of both extrudate-swell and tube-tooling die-extrusion coatings, using a hybrid finite element/finite volume discretisation *fe/fv*. Extrudate-swell presents a single dynamic free-surface, whilst the complex polymer melt coating flow exhibit two separate free-surface draw-down sections to model, an inner and outer conduit surface of the melt. The interest lies in determining efficient windows for process control over variation in material properties, stressing levels generated and pressure drop. In this respect, major rheological influences are evaluated on the numerical predictions generated of the extensional viscosity and Trouton ratio, when comparing solution response for an exponential Phan-Thien Tanner (EPTT, network-based) model to that for a single extended Pom-Pom (SXPP, kinematic-based) model. The impact of shear-thinning is also considered. Attention is paid to the influence and variation in Weissenberg number We , solvent-fraction β (polymeric concentration), and second normal stress difference N_2 (ξ parameter for both EPTT, and α anisotropy parameter for SXPP). The influence of model choice and parameters upon field response is described in situ through, pressure, shear and strain-rates and stress. The numerical scheme solves the momentum-continuity-surface equations by a semi-implicit time-stepping incremental Taylor-Galerkin/pressure-correction finite element method, whilst invoking a cell-vertex fluctuation distribution/median-dual-cell finite volume approximation for the first-order space-time hyperbolic-type stress evolution equation.

KEY WORDS:

Taylor-Galerkin, tube-tooling, cable-coating, die-extrusion, free-surface, exponential Phan-Thien Tanner model, single extended Pom-Pom model

1 INTRODUCTION

The modelling of annular extrudate-swell and cable-coating flows, with dynamic free-surfaces, remains a popular topic which has motivated many studies over recent years. In this context, extrudate- or die-swell has been adopted as a benchmark viscoelastic flow problem [1], characterised by specific features of: the presence of a sharp separation point at the exit of the die, the location and shape of the free-surface spawned, and the impact of viscoelasticity upon flow response (see for example [2–5]).

Moreover, there have been a number of studies that have addressed the modeling of cable-wire coating for tube-tooling (see for example [6–8]). These have provided some progress within the inelastic, non-isothermal and viscoelastic regimes. In their work, viscoelastic coating flows were simulated with a Phan-

Thien/Tanner (PTT) model and solved with a finite element technique. Based on, a time-stepping Taylor-Galerkin/pressure-correction finite element framework [9], tube-tooling extrusion coating with a low-density polymer melt has been analysed by Mutlu et al. [10]. In this study, and whilst addressing the coupling in the system between velocity and stress, both decoupled and coupled numerical approaches were adopted. The strain-hardening/softening, shear-thinning properties of the polymeric coatings were approximated with the exponential form of the Phan-Thien/Tanner (EPTT) model. There, the decoupled scheme was advocated to handle numerical solution instabilities at low-solvent concentrations representative for polymer melt response ($\beta \leq 10^{-3}$). Building upon this success, Matallah et al. [11] used the finite element method to simulate high-speed viscoelastic wire-coating, again employing coupled and decoupled schemes. There, predictions for a

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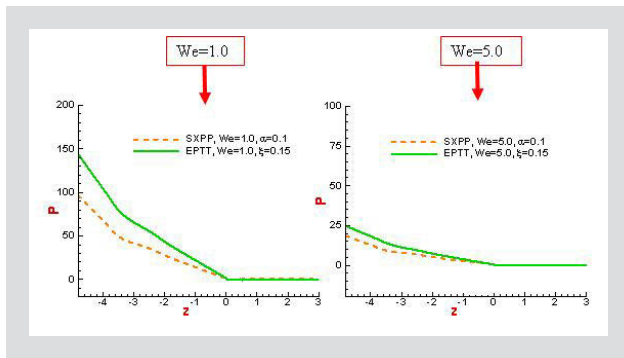


Figure 13: Pressure profiles, $SXPP\{q = 2, \epsilon_{SXPP} = 0.1, \alpha = 0.1\}$ versus $EPTT\{\epsilon_{EPTT} = 0.5, \xi = 0.15\}$ (We variation, $\beta = 10^{-3}$).

ity property). This has been accomplished here by fixing ϵ_{SXPP} and allowing q to vary. Here, a near-optimal least-squares fit has been extracted to experimental data, across the complete range of shear and extensional viscosity. Through the knowledge on molecular stretch, the SXPP model provides a way forward in practical applications where one seeks to take advantage of recovering structures in a complex flow.

Initially, the extrudate-swell benchmark problem with a dynamic free-surface has been studied, and then, subsequently extended into a study on the more complex industrial flow of tube-tooling cable-coating. The latter problem manifests two separate top and bottom moving free-surfaces. For the extrudate-swell problem, attention is paid to describe the impact of Weissenberg number We and solvent-fraction on swelling ratio. Findings reflect that, an increase in swelling occurs as We increases and β decreases. In addition, strain-rate stabilisation has also been investigated for this problem to interrogate the influence of singularity capturing on the die-exit solution. In this respect, it has been successfully demonstrated that such treatment can have a significant impact on the peak-level of stress exiting the die. In turn, this influences the accurate determination of free-surface profiles, where such variation has been detected in swelling-ratio at selected Weissenberg numbers. Moreover, the swelling ratio is also observed to decrease when some non-zero second-normal stress-difference is incorporated within the model representation.

Under tube-tooling flow with the EPTT and SXPP models, the respective influences of Weissenberg number We , solvent-fraction β , and second normal stress-difference N_2 (ξ_{EPTT} for EPTT, ξ_{SXPP} and α_{SXPP} for SXPP) have each been systematically investigated. Under shear-thinning properties, a decline in the total pressure-drop is observed as Weissenberg number We increases and solvent-fraction β decreases. Under We variation, a weak contribution of radial stress τ_{rr} arises, leading to minor adjustment in the normal stress τ_{zz} and first normal stress N_1 . Comparative data are presented for SXPP and EPTT solutions, governing total pressure-drop and stress production, both with and without N_2 contributions. Generally, lower levels of pressure-drop and stress are observed in SXPP as op-

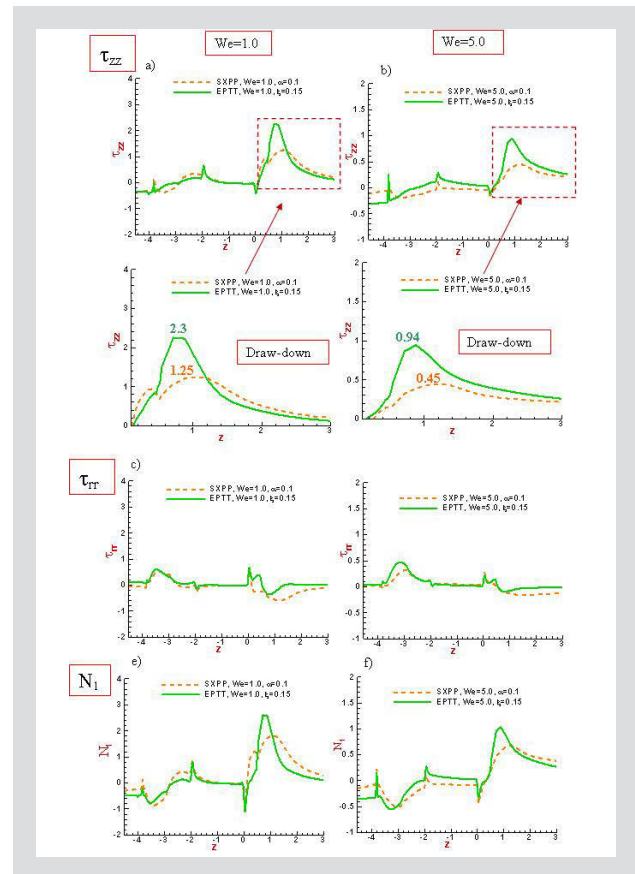


Figure 14: τ_{zz} , τ_{rr} and N_1 profiles at centreline, $SXPP\{q = 2, \epsilon_{SXPP} = 0.1, \alpha = 0.1\}$ versus $EPTT\{\epsilon_{EPTT} = 0.5, \xi = 0.15\}$ (We variation, $\beta = 10^{-3}$).

posed to EPTT predicted solutions, due to the exaggerated propensity of SXPP to thin at faster rate than for EPTT. Furthermore, a lower level of residual-stress N_1 is observed in the final coating at higher levels of elasticity: for example with EPTT, there is a drop of almost 30 % in peak-value from the $We = 5$ to $We = 20$ solution.

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