A Simplified Model for the Evaluation of the Rheological Properties of a Suspension of Solids in a Power-Law Fluid

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Abstract:
An effort is made to determine theoretically the new rheological properties of a rheologically defined non-Newtonian fluid as a result of the addition of particles in the original fluid. A theoretical model is proposed to determine the rheological properties of a dilute suspension of infinite-length round particles for a Power-Law fluid, which is treated as a homogeneous fluid with new rheological properties. The equations of a two-dimensional, dilatational, creeping, steady-state flow of a near-Newtonian fluid around a solid circular particle are developed and solved. The solution is obtained by computation of the dissipation relation and is based on the change of the shear rate in a shear flow of the suspension. The model is solved numerically. The resulting from the particle addition fluid is found to be more shear-thinning in comparison to the original. Experimental evidence from the literature supports the soundness of the present findings.

Zusammenfassung:
In dieser Arbeit wird ein Ansatz dargestellt, um das rheologische Verhalten eines definierten, mit Partikel gefüllten nicht-Newtonschen Fluids als Funktion der Partikelkonzentration zu bestimmen. Ein theoretisches Modell wird vorgestellt, um die rheologischen Eigenschaften einer verdünnten Suspension von unendlich langen, runden Partikeln in einem Potenzgesetz-Fluid zu ermitteln, das durch ein homogenes Fluid mit neuen rheologischen Eigenschaften modelliert wird. Die Gleichungen für eine zwei-dimensionale stationäre Kriechströmung in Dehnung um einen festen, kreisförmigen Partikel werden aufgestellt und gelöst. Die Lösung folgt aus der Berechnung der Dissipationsbeziehung und basiert auf einer Änderung der Schergeschwindigkeit in der Scherströmung der Suspension. Das Modell wird numerisch gelöst. Die Zugabe der Partikel führt zu einem ausgeprägten strukturviskosen Verhalten im Vergleich zu der reinen Flüssigkeit. Experimentelle Literaturdaten unterstützen unsere Ergebnisse.

Résumé:
Un effort pour déterminer théoriquement les nouvelles propriétés rhéologiques d'un fluide non Newtonien résultant de l'addition de particules au fluide original est présenté. Un modèle théorique est proposé afin de déterminer les propriétés rhéologiques d'une suspension diluée de particules rondes infiniment longues dans un fluide à loi de puissance, qui est traitée comme un fluide homogène possédant de nouvelles propriétés rhéologiques. Les équations d'un écoulement bidimensionnel dilatant et établi d'un fluide non newtonien autour d'une particule solide et circulaire sont développées et résolues. La solution est obtenue en calculant la relation de dissipation et est basée sur le changement de la vitesse de déformation dans l'écoulement en cisaillement de la suspension. Le modèle est résolu numériquement. Le fluide résultant de l'addition de particules s'avère plus rhéo-amincissant que le fluide originel. L'évidence expérimentale reportée dans la littérature supporte les résultats présentés.

Key words: Suspension, non-Newtonian, shear-thinning, rheological properties

1 INTRODUCTION

Non-Newtonian fluids play an important role in various industries. They are employed in food, cosmetics, chemical, aerospace [1] and other industries. Some non-Newtonian fluids exhibit shear-thinning behavior and have a finite yield stress, properties that make such fluids attractive for many applications. Their significant advantage is the possibility of addition of powders to the fluid. In Newtonian liquids, powders tend to settle in the bottom of the tank during storage and render the addition useless. Non-Newtonian fluids have high, controllable zero-shear viscosity and yield stress, thus reducing the

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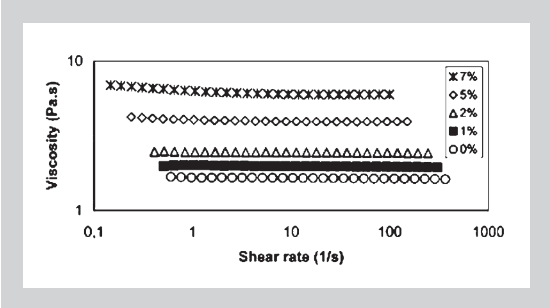

Figure 5: The experimental results of Harzallah and Dupuis [14] for suspensions in a Newtonian fluid.

tankar and Hu [27] perform a theoretical assessment of a shear flow of such a suspension and support our findings. Examination of the derivative of the shear stress in Figure 3 of Reference 27 clearly indicates that shear-thinning was obtained. In another paper [18], Patankar and Hu show that dilute suspensions in a non-shear-thinning fluid exhibit shear-thinning behavior.

The second reason why theoretical works in many cases do not find non-Newtonian behavior is due to the order of magnitude of these effects. In this work, we consider effects of o(ϕ), which are perturbations on the Newtonian solution. In turn, the Newtonian solution is a perturbation of the undisturbed flow. Therefore, the flow due to non-Newtonian effects is two orders of magnitude smaller than the undisturbed flow. Most works neglect these orders of magnitude [3, 5, 28]. However if these orders of magnitude are not neglected, non-Newtonian effects appear [27, 29]. The non-Newtonian effects are so weak that for any practical purpose, a dilute suspension of round particles in a Newtonian fluid can be considered Newtonian and Einstein's result (Equation 2) is valid. Therefore, the importance of the results lies in the theoretical sphere, since (as it is shown below) it explains the onset of non-Newtonian behavior that can be observed in more concentrated suspensions.

There are also some experimental works that support our findings. Most of the experimental work is done on three-dimensional bodies; however, it changes only the quantitative result. The principle that a dilute suspension of round particles in a Newtonian fluid exhibits a shear-thinning behavior stands. Dullaet and Mewis [30] present the results of rheological measurements of a 2.9 % fumed silica dispersion in a Newtonian mix of paraffin oil and polyisobutylene. From their results it is evident that the suspension exhibits a slight shear-thinning behavior. The Harzallah and Dupuis [14] results, presented in Figure 5, clearly show a slight shear thinning for suspensions of concentration as low as 5 % (it also might have been seen at 2% concentration, but the authors show large symbols for their plot and the trend is not clear). Several series of experimental observations of suspensions in Newtonian fluids, for concentrations from 11.8 to 15 %, show that mild shear-thinning is obtained [10–12]. In the case of an 11.8 % suspension [10] the resulting power index was $n_s = 0.88$, which is in line with the trend found in this work.

Another issue is about what is not found in the literature. As it was mentioned in the introduction, it is observed that at relatively low shear-rates non-dilute suspensions exhibit a shear-thinning behavior. However, there is no accepted concentration for that which it happens. The findings in this work may indicate that non-Newtonian behavior starts immediately and is always present, but is not always visible. A possible answer to this has to do with the nature of shear-thinning fluids. In general, the viscosity is expected to decrease by several orders of magnitude with increasing shear rate, whereas the presented theory, valid for a dilute, near-Newtonian fluid, predicts only a small drop in the measured viscosity. As was explained above, the order of magnitude of non-Newtonian effects is very small. Therefore, it can be easily overlooked or be treated as experimental uncertainty or small instrument error.

6 CONCLUSIONS

A viscosity analysis has been conducted for dilute suspensions of near-Newtonian, P-L suspending fluids. New rheological indexes and new viscosity of the resulting fluid have been calculated. It has been shown that the lower the P-L exponent of the suspending fluid is (i.e., the fluid is more shear-thinning), the resulting fluid is even more shear-thinning (P-L exponent decreases) and its viscosity is higher. It also has been found that the shear-thinning behavior of suspensions based on Newtonian fluids starts even when the suspensions are dilute. This result requires more investigation but it lays down a theoretical basis for the non-Newtonian behavior of suspensions.

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REFERENCES

[1] Natan B, Rahimi S: The status of gel propellants in year 2000, Intern. J. Energetic Materials Chem. Propuls. 5 (2002) 172-194.

[2] Macosko CW: Rheology Principles, Measurements and Applications, Wiley-VCH, Inc. (1994).

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http://www.appliedrheology.org

[3] Einstein A: A new determination of molecular dimensions, Annalen der Physik 19 (1906) 289-306, correction in Annalen der Physik 34 (1911) 591-592.
[4] Batchelor GK: An Introduction to Fluid Dynamics, Syndics of the Cambridge University Press, Cambridge, Great Britain (1967) 246-254.
[5] Probstein RF: Physicochemical Hydrodynamics: An Introduction, John Wiley & Sons, Inc. (1994).
[6] Thomas DG: Transport characteristics of suspensions: VIII. A note on the viscosity of Newtonian suspensions of uniform spherical particles, J. Colloid Interface Sci. 20 (1965) 267-277.
[7] Thomas CU, Muthukumar M: Three-body hydrodynamic effects on viscosity of suspensions of spheres, J. Chem. Phys. 94 (1991) 5180-5189.
[8] Hsue CH, Becher PF: Effective viscosity of suspensions of spheres, J. Am. Ceram. Soc. 88 (2005) 1046-1049.
[9] Stickel JJ, Powell PL: Fluid mechanics and rheology of dense suspensions, Ann. Rev. Fluid Mech. 37 (2005) 129-149.
[10] Yoo B, Rao MA: Effect of unimodal particle size and pulp content on rheological properties of tomato puree, J. Texture Stud. 25 (1994) 421-436.
[11] Heymann L, Peukert S, Aksel N: On the solid-liquid transition of concentrated suspensions in transient shear flow, Rheol. Acta 41 (2001) 307-315.
[12] Usui HK, Kishimoto K, Suzuki H: Non-Newtonian viscosity of dense slurries prepared by spherical particles, Chem. Eng. Sci. 56 (2001) 2979-2989.
[13] Bharucha VF: Rheological study of hydroxypropyl guar (hpg) slurries, Master's thesis, The Mewbourne School of Petroleum and Geological Engineering, Graduate College, University of Oklahoma (2004).
[14] Harzallah OA, Dupuis D: Rheological properties of suspensions of TiO2 particles in polymer solutions. 1. Shear viscosity, Rheol. Acta 42 (2003) 10-19.
[15] Bartels N, von Kampen J, Ciezki HK, Zanetti N: Investigation of the spray characteristics of an aluminized gel fuel with an impinging jet injector, Proc. 35^{th} Int. Annual Conference of ICT, Karlsruhe (2004) 17.1-17.12.
[16] Greco F, D'Avino G, Maffettone PL: Rheology of a dilute suspension of rigid spheres in a second order fluid, J. Non-Newtonian Fluid Mech. 147 (2007) 1-10.
[17] Housiadasa KD, Tanner RI: On the rheology of a dilute suspension of rigid spheres in a weakly viscoelastic matrix fluid, J. Non-Newtonian Fluid Mech.162 (2009) 88-92.
[18] Patankar NA, Hu HH: Rheology of a suspension of particles in viscoelastic fluids, J. Non-Newtonian Fluid Mech. 96 (2001) 427-443.
[19] Hwang WR, Hulsen MA, Meijer HEH: Direct simulations of particle suspensions in a viscoelastic fluid in sliding bi-periodic frames, J. Non-Newtonian Fluid Mech.121 (2004) 15-33.
[20] Lee BJ, Mear ME: Effective properties of power-law solids containing elliptical inhomogeneities. Part I: Rigid inclusions, Mech. Mater. 13 (1992) 313-335.
[21] Leal LG: Particle motions in a viscous fluid, Ann. Rev. Fluid Mech. 12 (1980) 435-476.
[22] Speziale CG: On the advantages of the vorticity-velocity formulation of the equations of fluid dynamics, J. Comput. Phys. 73 (1987) 476-480.
[23] Bird RB, Stewart WE, Lightfoot EN: Transport Phenomena, John Wiley & Sons, Inc. (2002)
[24] Nicomedo L, Nicolais L, Landel RF: Shear rate dependent viscosity of suspensions in Newtonian and non-Newtonian liquids, Chem. Eng. Sci. 29 (1974) 729-735.
[25] Manley RStJ, Mason SG: The viscosity of suspensions of spheres: A note on the particle interaction coefficient, Can. J. Chem. 32 (1954) 763-767.
[26] Krieger I: Rheology of monodisperse lattices, Adv. Colloid Interface Sci 3 (1972) 111-136.
[27] Patankar NA, Hu HH: Finite Reynolds number effect on the rheology of a dilute suspension of neutrally buoyant circular particles in a Newtonian fluid, Int. J. Multiphase Flow 28 (2002) 409-425.
[28] Batchelor GK: The stress system in a suspension of force-free particles, J. Fluid Mech. 41 (1970) 545-570.
[29] Batchelor GK, Green JT: The determination of the bulk stress in a suspension of spherical particles to order c^2, J. Fluid Mech. 56 (1972) 401-427.
[30] Dullaet K, Mewis J: Stress jumps on weakly flocculated dispersions: Steady state and transient results, J. Colloid Interface Sci. 287 (2005) 542-551.

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