Interpreting Shear Creep Data for Bread Dough Using a Damage Function Model

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Abstract:
We have interpreted the results of shear creep data on samples of bread dough, tested in a parallel plate rheometer, by using a damage function model. Whilst the agreement between calculation and experimental results is satisfactory for the dough for stress levels less than 500 Pa, increasingly large deviations from the predictions occur for stress levels of 500 and 1000 Pa. This is in contrast with the behaviour in simple shearing, where agreement with the damage function model can be obtained up to shear stresses of several kPa. It is therefore of interest to see why the discrepancy between model predictions and experiments occurs in shear creep at such low stress levels. It is shown that edge fracture in a parallel-plate rheometer, due to the second normal stress difference, N_2, is responsible for the deviations and the model behaves quite well for stress levels 300 Pa and below, where edge fracture is not important. Therefore the edge fracture instability, which depends on N_2, limits the range of stress which can be applied in shear creep tests.

Zusammenfassung:
Wir interpretieren Kriechversuche an Brotteig in einer Parallele Platten-Geometrie durch ein sogenanntes „damage-Funktionsmodell". Die Übereinstimmung zwischen Theorie und Experiment ist nur für Scherspannungen kleiner als 500 Pa befriedigend. Bei größeren Spannungen (500 und 1000 Pa) erhält man große Abweichungen. Dieses Verhalten kontrastiert mit Scherversuchen, wo man eine Übereinstimmung bis in den Bereich von einigen kPa findet. Daher ist es interessant, diese Abweichungen zu erklären. Wir zeigen, dass ein Randbruch im Teig wegen der zweiten Normalspannungsdifferenz (N_2) auftritt. Der Randbrucheffekt begrenzt die Scherspannung in Kriechversuchen um etwa 300 Pa.

Résumé:
Les résultats expérimentaux de fluage en cisaillement réalisés sur des échantillons de pâte à pain ont été interprétés par le biais d'un modèle d'endommagement. Alors que l'accord entre théorie et résultats expérimentaux est satisfaisant pour les contraintes inférieures à 500 Pa, d'importants écarts apparaissent pour des niveaux de contraintes de 500 et 1000 Pa. Ceci contraste avec le comportement en cisaillement simple pour lequel un bon accord peut être obtenu avec le modèle d'endommagement pour des contraintes allant jusqu'à plusieurs kPa. Il est donc intéressant d'étudier l'origine des différences entre les prédictions du modèle et les résultats expérimentaux à de si faibles niveaux de contraintes. Il apparait que les instabilités de surface au niveau du bord libre de l'échantillon observées dans un rhéomètre plan-plan et dues à la seconde différence de contraintes normales (N_2) sont responsables de ces écarts et le modèle produit de bons résultats pour des contraintes inférieures à 300kPa, pour lesquelles les instabilités sont faibles. Ainsi, les instabilités de surface qui dépendent de N_2, limitent le domaine de contraintes qui peuvent être appliquées au fluage en cisaillement.

Key words: bread dough, shear creep, damage function, edge fracture

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$$N_2 = -aN_1 \quad (17)$$

with a = 0.038, p = 0.3 (Tanner et al. [4]) we find $|N_2| \sim 0.03\,\tau\gamma$. According to Keentok and Xue [11] edge fracture depends on N_2, the gap h at the rim and the surface tension coefficient σ at the free edge of the sample at $r = R$. Their criterion for edge fracture is that

$$|N_2| > \frac{5.5\sigma}{h} \quad (18)$$

where σ is the surface tension coefficient, here assumed to be that of water ($\sigma \sim 0.07$ kg/s²), and h = 0.002 m (It is likely that the relevant surface tension is somewhat lower, due to the petroleum jelly layer). Hence fracture occurs, using Equations 17 and 18, if a critical shear γ_c of

$$\gamma_c \sim \frac{6400}{\tau} \quad (19)$$

is applied. Here both γ_c and τ are the true values at the rim of the instrument. Table 2 shows the critical shear as a function of shear stress τ (Pa). In the Table, the shear stresses and strains refer to the rim conditions; the nominal values at r = 3/4 R used in the earlier part of the paper are exactly 3/4 of this strain and slightly more than 3/4 of the stress (due to the variation in f with strain). The usual fracture strain foe this dough is around 20 [4], so the edge fracture gives a premature failure if the γ_c valuess in table 2 are less than 20. Thus in terms of the nominal values one expects to see edge fracture for the two largest stresses in Table 2, but not for the lower three. For rim stresses of 500 and 1000 Pa one sees consequent increased plate rotation speeds. If a lower surface tension coefficient were assumed, then γ_c (in Table 2) would be correspondingly lower.

6 CONCLUSION

Clearly, the edge fracture problems encountered at higher stresses dominate the creep response of the dough and limit the stress which can be applied to around 300 Pa. Aside from these cases, the damage function model behaves with reasonable accuracy. The key factor in the model leading to edge fracture is the non-zero N_2 predicted by the imposed strain measure: Without N_2 (a = 0) no edge fracture of the kind postulated is predicted. Also although the N_2 factor does not appear in the shear stress calculation, it was found necessary when describing biaxial deformation [4].The edge fracture limits shear creep testing to lower shear stress values. It seems curious that a constant shear rate test permits higher shear stress values to be reached. An explanation is that in simple shearing (beginning at t = 0) the build-up of stress is gradual and hence the onset of a large N_2 and consequent edge fracture is delayed: In shear creep testing the full stress is applied at t = 0, and rapid edge fracture occurs.

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APPENDIX

In the spirit of Shaw and Liu [7] we investigate the parallel-plate response with the dough model. We can rewrite the f-function in terms of $\ln\gamma$, instead of $\log\epsilon_H$ by using the connection between ϵ_H and γ (Equation 13). Doing this gives, for JANZ dough

$$f = 0.2193 - 0.1043\ln\gamma, \qquad \gamma \le 2.65 \quad (A1)$$

$$f = 0.1704 - 0.0503\ln\gamma, \qquad \gamma > 2.65 \quad (A2)$$

Thus we have two regions with f of the form $d + b\ln\gamma$. Now the shear stress τ at radius r with a steady shear rate ($\Omega r/h$) with a gap h is

$$\tau = f(\gamma)\int_0^t G(t-t')\frac{\Omega r}{h}dt' = fG(1)\frac{\Omega r}{h(1-p)}t^{1-p} \quad (A3)$$

where we have set $G(t) = G(1)t^{-p}$, and $G(1)$ and t are constants: $\gamma = \Omega rt/h$ at that radius. The moment M on the plate is

$$M = 2\pi\int_0^R \tau r^2 dr \quad (A4)$$

τ [Pa]	γ_c
10	640
100	64
200	32
500	12.8
1000	6.4

Table 2: Critical shear values (rim values).

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or

$$M = 2\pi G(1)\frac{\Omega t^{1-p}}{h(1-p)}\int_0^R f(\gamma)r^3 dr \qquad \text{(A5)}$$

To evaluate the integral we set

$$\int_0^R f(\gamma)r^3 dr = \frac{h^4}{\Omega^4 t^4}\int_0^{\gamma_0} f(\gamma)\gamma^3 d\gamma \qquad \text{(A6)}$$

If f is of the form above, $f = d + b\ln\gamma$, we find

$$\tau^* = \frac{3M}{2\pi R^3} = \frac{3G(1)\gamma_0 t^{-p}}{4(1-p)}\left(d + b\ln\gamma_0 - \frac{b}{4}\right) \qquad \text{(A7)}$$

τ^* is the approximate value of the shear stress derived for the Equation 3 above: In terms of $\gamma = 3\gamma_0/4$ we have

$$\tau^* = \frac{G(1)t^{-p}}{(1-p)}\gamma\left(d + b\ln\gamma - \frac{b}{4} + b\ln\frac{4}{3}\right) \qquad \text{(A8)}$$

whereas the true value of τ is

$$\tau = \frac{G(1)t^{-p}}{(1-p)}\gamma(d + b\ln\gamma) \qquad \text{(A9)}$$

Hence the error, $(\tau-\tau^*)/\tau = 1-\tau^*/\tau$ is given by

$$1 - \frac{\tau^*}{\tau} = -\frac{b\left(\ln\frac{4}{3} - 0.25\right)}{d + b\ln\gamma} = -\frac{0.0377b}{d + b\ln\gamma} \qquad \text{(A10)}$$

Since $|b|$ is about 0.1043, or 0.0503, the errors are small, e.g. in the order of 1%, except for large strains γ (~ 15) where they reach ~5 %. Hence here we have used the simple rule Equation 6: Considering possible experimental errors, we believe this adequate (If the reference radius were change to 0.778 for 0.75, in fact the result A(8) would be exact for the JANZ material).

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