# Onset of Instabilities in Taylor-Dean Flow of Yield-Stress Fluid

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**Abstract:**
We present an experimental study of the stability of the Taylor-Dean flow of well characterized suspensions of solid disks occurring between two horizontal coaxial cylinders. The inner cylinder is rotating and the outer cylinder is at rest. By means of a visualization technique, we determine the shape of the vortices which take place in the flow at the onset of the instability and the corresponding critical parameters against the flake concentration and the system aspect ratio.

**Zusammenfassung:**
Dieser Artikel stellt eine experimentelle Studie der Taylor-Deam-Strömung gut charakterisierter Plattensuspensionen vor, die zwischen zwei koaxialen, waagerechten Zylindern hervorgerufen wird. Der innere Zylinder ist in Drehung, während die Außenseite ruht. Mit Hilfe einer Visualisierungs-Methode bestimmen wir die Form der Luftwirbel, die im Abfluss entstehen, sowie die entsprechend kritischen Parameter gemäß der Konzentration der Platten und der Geometrie des Systems.

**Résumé:**
Cet article présente une étude expérimentale de l'écoulement de Taylor-Dean de suspensions de disques bien caractérisées, généré entre deux cylindres horizontaux et coaxiaux. Le cylindre intérieur est en rotation alors que celui extérieur est fixe. A l'aide d'une technique de visualisation, nous déterminons la forme des tourbillons qui prennent naissance dans l'écoulement ainsi que les paramètres critiques correspondants en fonction de la concentration des disques et du rapport d'aspect du système.

**Key words:** Baroclinic instability, Herschel-Bulkley model, necking cell, suspension, Taylor-Dean instability, Taylor vortices, wavy Taylor vortices

## 1 INTRODUCTION

Dean [1] considered the flow of a viscous fluid in a curved channel under a pressure gradient. He showed that there occurs an instability of a similar type to the Taylor vortex flow. The Taylor-Dean flow is the combination of the Taylor-Couette flow (generated between two coaxial rotating cylinders), [2] with the Dean flow (the fluid being pumped at the same time around the annulus). The Taylor-Dean arrangement is concerned with many engineering applications such as paper making [3]. Raney and Chang [4] (see also Drazin and Reid [5]) showed that there exists oscillatory marginal modes and stationary three-dimensional ones in the Taylor-Dean problem. The theoretical study was performed by Diprima [6]. Generally, the authors assumed the onset of the instability to be axisymmetric and stationary. Chen and Chang [7] presented a linear stability parametric study of the Taylor-Dean flow in a wide range of the two parameters governing the problem. These parameters are the ratio of angular velocity of the outer cylinder to that of the inner cylinder and the ratio of pumping and rotation velocity denoted here by $\lambda$. They furnished a topology of the nature of the instability. So they





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Moreover we note an increasing of the value of *Ta* according to the concentration for all values of the aspect ratio Γ'. Analyzing these data, we can deduce an experimental law from the experimental curves as

$$Ta = a + b^{c\Gamma'} \tag{7}$$

The values of the constants *a*, *b* and *c* are determined by fitting from the experimental curves (Tables 1 and 2).

## 4 CONCLUSIONS

We performed an experimental investigation of the effect of the onset of hydrodynamic instability in the Taylor-Dean flow of suspensions of solid disks of Aluminium in a Simili gazoline-Challala M mixture (80/20). The suspensions were well characterized. They have a viscoplastic rheological behaviour following the Herschel-Bulkley equation of state. The stability of any rotating hydrodynamic system has never been considered before for this rheological model. The variation of apparent viscosity and rheological parameters of the suspension against the disk concentration *C* was then derived from the previous rheogramms. The apparent viscosity, the consistency and the power law index follow a linear dependence on *C*, while the yield stress remains quite constant with the same average value as for the suspending liquid.

The onset of the instability was then investigated using a visualization technique. For given concentration in the range studied, the nature of the structures taking place in the flow system depends on the radial aspect ratio tested, while for given radial aspect ratio in the range studied, the value of the critical Taylor number characterizing the onset of the instability in the flow system depends on the particle concentration.

We notably found that the evolution of the value of critical Taylor number Ta corresponding to the occurrence of the first instability is nonlinear according to the radial aspect ratio. First of all, we noted that for high radial aspect ratio, namely $10.16 < \Gamma' < 10.44$, the secondary motion which is superimposed on the base Couette flow is of (well known) stationnary Taylor vortex flow type. The flow stability being analogous to that (well known) of a closed system, we could focus our investigations on radial aspect ratio such that $\Gamma' < 10.16$, and we notably identified three regions. In the range $10.16 < \Gamma' < 9.85$ we noted a stabilizing effect of the flow according to the radial aspect ratio. This is due to the effect of the free surface. Furthermore, we noted the occurrence of a necking cell at the exit region. In the second range $8.22 < \Gamma' < 9.85$, a source of instability occurs, which is located at the extremity of the flow system generating an inclined travelling rolls with the coexistence of a baroclinic instability, which itself is responsible for the decreasing of the critical Taylor number characterizing the onset of the instability in the flow system. Finally, in the third range $2.19 < \Gamma' < 8.22$, the instability has the form of inclined travelling rolls. Moreover, the baroclinic instability does no longer exist for $\Gamma' < 7.82$ inducing an increasing of the Taylor number according to the decreasing of Γ', which indicates a stabilizing effect of the free surface. We also noted an increasing of critical Taylor number according to the concentration, which indicates a stabilizing effect of the particle as described by an exponential evolution for the second and the third regions.

## REFERENCES


[1] Dean W: Fluid motion in a curved channel, Proc. Roy. Soc. Lond., Ser. A121 (1928) 402-420.

[2] Taylor GI: Stability of a viscous liquid contained between two rotating cylinders, Philos. Trans. Roy. Soc. Lond. Ser. A233 (1923) 289-343.

[3] Brewster DB and Nissan AH: The hydrodynamics of flow between horizontal concentric cylinders, Chem. Eng. Sci. 7 (1958) 215-221.

[4] Raney DC and Chang TS: Oscillatory modes of instability for flow between rotating cylinders with a transverse pressure gradient, Z. Angew. Math. Phys. 22 (1971) 681-690.

[5] Drazin PG and Reid WH: Hydrodynamic stability, Cambridge University Press, Cambridge (1982).

[6] Diprima RC: The stability of viscous flow between rotating cylinders with a pressure gradient acting around the cylinders, J. Fluid Mech. 6 (1959) 462-468

[7] Chen F and Chang MH: Stability of Taylor-Dean flow in a small gap between rotating cylinders, J. Fluid Mech. 243 (1992) 443-455.

[8] Mutabazi I, Hegseth JJ and Andereck CD: Spatiotemporal pattern modulations in the Taylor-Dean system, Phys. Rev. Lett. 64 (1990) 1729-1732.

[9] Mutabazi I, Hegseth JJ, Andereck CD and Wesfreid, JE: Pattern formation in flow between two horizontal coaxial cylinders with a partially filled gap, Phys. Rev., A38 (1988) 4752-4760.







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[10] Mutabazi I, Wesfreid JE, Hegseth JJ and Andereck CD: Recent experimental results in the Taylor-Dean system, Eur. J. Mech. B/Fluids 10 (1991) 239-245.
[11] Joo YL and Shaqfeh ESG: Viscoelastic Poiseuille flow through a curved channel: A new elastic instability, Phys. Fluids A3 (1991) 1691-1694.
[12] Joo YL and Shaqfeh ESG: A purely elastic instability in Dean and Taylor-Dean flow, Phys. Fluids A4 (1992) 524-543.
[13] Joo YL and Shaqfeh ESG: Observations of purely elastic instability in the Taylor-Dean flow of a Boger fluid, J. Fluid Mech. 262 (1994) 27-73.
[14] Ould Elaya MM and Gunasekaran S: Rheology of sulfate suspensions and pre-thickened beverages used in diagnosis and treatment of dysphagia, Appl. Rheol., 17 (2007) 33137-1-33137-8.
[15] Loret C, Frith W and Fryer PJ: Mechanical and structural properties of Maltodextrin/Agarose microgels composites, Appl. Rheol., 17 (2007) 31412-1-31412-13.
[16] Skali S.: Contribution à l'étude de l'écoulement de la pâte à papier: Interaction floculation-turbulence, Ph.D. Nancy (1991).
[17] Jankowski DF and Takeuchi DI: The energy stability limit for flow in a curved channel, Trans. ASME E: Appl. Mech. 43 (1976) 548-550.
[18] Nsom B: Stability of Taylor-Dean flow of fiber suspensions, Int. J. Appl. Mech. Eng. 3 (1998) 185-204.
[19] Ericksen JL: Transversely isotropic fluids, Z. Kolloid 173 (1960) 117-122.
[20] Watson JH: The diabolical case of the recurring yield stress, Appl. Rheol. 14 (2004) 40-45.
[21] Loret C, Frith W and Fryer PJ: Mechanical and structural properties of Maltodextrin/Agarose gels composites, Appl. Rheol. 16 (2006) 248-257.
[22] Kelessidis VC and Maglione R: Shear Rate Corrections for Herschel-Bulkley Fluids in Couette Geometry, Appl. Rheol. 18 (2008) 34482-1-34482-11.
[23] Graebel PG: The hydrodynamic stability of a Bingham fluid in Couette flow, Proceedings of the International Symposium on Second-Order Effects in Elasticity, Plasticity and Fluid Dynamics, Haifa (1964) 636-639.
[24] Peng JIE and Zhu K: Linear stability of Bingham fluids in spiral Couette flow, J. Fluid Mech. 512 (2004) 21-45.
[25] Landry MP, Frigaard IA and Martinez DM: Stability and instability of Taylor-Couette flow of a Bingham fluid, J. Fluid Mech. 560 (2006) 321-353.
[26] Persoz B: Introduction à l'étude de la Rhéologie, Dunod, Paris (1960).
[27] Feys D, Verhoeven R and De Schutter G: Fresh self-compacting concrete, a shear-thickening material, Cem. Conc. Res. 38 (2008) 920-929.
[28] Feys D, Verhoeven R and De Schutter G: Evaluation of time independent rheological models applicable to fresh self-compacting concrete, Appl. Rheol. 17 (2007) 56244-1-56244-10.
[29] Chen KS, Ku AC, Chan TM and Yang SZ: Flow in the half-filled annulus between horizontal concentric cylinders in relative rotation, J. Fluid Mech. 213 (1990) 149-169.
[30] Ait Aider A, Skali S and Brancher JP: Visualisation de l'écoulement de Taylor-Dean ouvert, C. R. Mecanique, 333 (2005) 197-203.





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