UNSTEADY PARALLEL FLOWS OF AN ELASTO-VISCO-HYPOPLASTIC FLUID WITH OSCILLATING BOUNDARY

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Received: 1.11.2007, Final version: 3.8.2008

ABSTRACT:
In the present study, an evolution equation for the Cauchy stress tensor is proposed to take elastic, viscous and plastic characteristics of complex fluids simultaneously into account. In particular, hypoplasticity is incorporated to account for the plastic features. The stress model is applied to investigate time-dependent flows of an elasto-visco-plastic fluid driven by an oscillating boundary with/without an additional stationary boundary to study the cyclic responses and the model performance. Numerical simulations show that while different degrees of elastic and viscous effects can be captured by varying the model parameters, plastic deformation plays a significant role in the velocity distribution, and can be simulated appropriately by use of hypoplasticity. The stress model is capable of accounting for the combined elastic, viscous and plastic features of complex materials in transient motions, and applications may be found in geomorphic fluid motions like granular and debris flows, and flows involving polymers.

ZUSAMMENFASSUNG:
In dieser Arbeit wird eine Entwicklungsgleichung für den Cauchy-Spannungstensor vorgeschlagen, um gleichzeitg elastische, viskose und plastische Eigenschaften komplexer Fluide zu berücksichtigen. Insbesondere wird Hypoplastizität zur Berücksichtigung der Plastizität miteinbezogen. Dieses Spannungsmodell wird angewendet, um zeitabhängige Strömungen eines elasto-visko-plastischen Fluids zu untersuchen, die durch einen oszillierenden Rand mit/ohne zusätzliche stationäre Randbedingungen verursacht werden, um die periodische Antwort und die Modelleigenschaften zu erfassen. Numerische Simulationen zeigen, dass plastische Deformation eine wesentliche Rolle in der Geschwindigkeitsverteilung spielt und auf geeignete Weise durch die Verwendung der Hypoplastizität simuliert werden kann, während unterschiedliche Grade elastischer und viskoser Effekte durch Variation der Modellparameter erfasst werden können. Dieses Spannungsmodell kann gleichzeitig elastische, viskose und plastische Eigenschaften komplexer Materialien in transienten Bewegungen beschreiben. Anwendungen bestehen in geomorphologischen Fluidströmungen (z. B. Strömungen granulärer Materialien oder von Schutt) sowie in Strömungen von Polymeren.

RÉSUMÉ:
Dans cette étude, une équation évolutive pour le tenseur de contrainte de Cauchy est proposée afin de tenir compte simultanément des caractéristiques élastiques, visqueuses et plastiques des fluides complexes. En particulier, l'hypoplasticité est incorporée pour décrire les propriétés plastiques. Ce modèle de contrainte est utilisé pour examiner les dépendances en fonction du temps d'écoulements d'un fluide élasto-visco-plastique mis en mouvement par une limite oscillante avec ou sans limite stationnaire supplémentaire, dans le but d'étudier les réponses cycliques et les performances du modèle. Les simulations numériques montrent que, tandis que différents degrés d'effets élastiques et visqueux peuvent être décrits en variant les paramètres du modèle, la déformation plastique joue un rôle important sur la distribution des vitesses et peut être simulée correctement en utilisant l'hypoplasticité. Le modèle de contrainte est capable d'expliquer la combinaison des caractéristiques élastique, visqueuse et plastique des matériaux complexes en mouvement transitoire, et peut trouver des applications pour la description des déplacements des fluides géomorphiques comme les écoulements granulaires et de débris, et des écoulements concernant des polymères.

KEY WORDS: stress tensor; hypoplasticity; time-dependent flow

© Appl. Rheol. 18 (2008) 45001-1 – 45001-11

tion), the fluid immediately above the boundary moves with the same direction of the boundary due to the viscous effect. However, fluids above this layer hold the previous motion direction due to the plastic deformation. These give rise to the more convex velocity profiles near the oscillating boundary, when compared with Figure 5a. Such a tendency is also relevant in the time series of the velocity profiles at the middle of the channel for variations of the values of β_4 and β_5, as shown in Figure 8b. When β_4 and β_5 increase, plastic deformation above the oscillating boundary becomes stronger (and hence the velocity profiles there become sharper), resulting in larger amplitude of the velocity profiles at the center of the channel. However, due to the plastic deformation the phase lag only increases slightly, when compared with Figure 6b.

In short, for the flow above an infinite oscillating plate constrained by a parallel upper stationary plate, the penetration of the influence of the oscillating boundary depends on the distance between the two plates and the boundary layer thickness. As similar to the results shown in Section 2, the transmission of the shear is most efficient by the viscous effect, and subsequently by the hypoplastic and elastic effects. However, the phase lag of the velocity profile follows the same sequence in a reverse tendency.

4 CONCLUDING REMARKS

In the present study, an evolution equation for the Cauchy stress tensor was proposed to account for the combined elastic, viscous and plastic effects of a complex fluid simul-taneously. In particular, hypoplasticity was incorporated into the present model for the plastic effects due to its better features in distinguishing the elastic and inelastic range of deformation and in accompanying the loading and unloading history automatically. The model was subsequently applied to study the transient behaviour of a fluid above an oscillating plate with/without an upper stationary solid boundary. To this end, an explicit scheme was employed to solve the emerging two-point IBV problems numerically for better numerical convergence and stability. The main conclusions are summarized in the following:

- For unsteady parallel flows the shear generated on the oscillating boundary can better be transmitted toward the fluids when the elastic, viscous and plastic features of the fluids are taken into account for simultaneously. The penetration of the influence of the moving boundary is enhanced significantly in the present model than in the Newtonian model. Such an influence can be recognized by higher values of the boundary layer thickness.
- When elastic effect increases, the fluid needs a longer time to adjust itself to the variation of the oscillating boundary. The most part of the shear energy is preserved in the region near the moving boundary. These give rise to the convex velocity profiles near the oscillating plate, and the velocity profiles with smaller amplitude and larger phase lag in the upper part of the flow field.
- As viscosity increases, the adhesion between different layers of fluids and the solid boundary increases correspondingly. As a result, the shear can be transmitted most efficiently, and fairly “straight” velocity profiles across the flow field are obtained. Since the viscous effect is dominant, the fluid can response to the variation of the external excitation (here the oscillating plate) in a faster way, resulting smaller phase lag of the velocity profiles.
- By higher hypoplastic effect, plastic deformation occurring near the oscillating boundary is enhanced. This yields more convex velocity profiles near the oscillating plate in comparison with those from the cases of higher elastic effects, and are in particular visible when the oscillating boundary suddenly changes it motion direction. The amplitude of the velocity profiles in the upper part of the flow field increases as the hypoplastic effect increases; however, the phase lag only increases slightly due to the enhanced plastic deformation near the oscillating plate.

ACKNOWLEDGMENT

The authors are indebted to the National Science Council, Taiwan, for the financial support through the project NSC 95-2218-E-006-054, and the editors and referees for their detailed reviews which led to improvements.

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