Error Introduced by a Popular Method of Processing Parallel-Disk Viscometry Data

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Abstract:
The assumptions implicit in the simplified expressions used to convert the torque-rotational speed data of parallel-disk viscometry into rim shear rate and rim shear stress are identified. The rim shear stress generated by the simplified expression is compared against the actual rim shear stress. The error involved is quantified for two standard rheological models and for a set of laboratory data. Under normal operation conditions of parallel-disk viscometers this error was found to be within the acceptable limit. However, for highly shear thinning fluids and for fluids exhibiting yield stress this error can become very large. The suitability of the approximate rim shear stress in wall slip determination is then briefly discussed.

Zusammenfassung:
Die Annahmen, die zu den vereinfachten Gleichungen führten, für die Umrechnung der Geschwindigkeitsdaten des Drehmoments aus der Parallelscheibenviskosimetrie in Randscherrate und Randscherspannung umzurechen. Die durch den vereinfachten Ausdruck berechnete Randscherspannung wird mit ihrem tatsächlichen Wert verglichen. Für eine Reihe von rheologischen Modellen sowie eine Auswahl von Datensätzen quantitativ wird der entstehende Fehler bestimmt. Es wurde festgestellt, dass dieser unter Normbedingungen für die Parallelscheibenviskosimetrie innerhalb eines akzeptablen Bereichs liegt. Für hoch strukturviskose Flüssigkeiten und für Flüssigkeiten, die eine Streckgrenze aufweisen, wurde dieser Fehler jedoch sehr groß. Im Anschluss folgt eine kurze Diskussion über die Tauglichkeit der genäherten Randscherspannung zur Bestimmung des Wandgleitverhaltens.

Résumé:
Les hypothèses implicites, dans les expressions simplifiées utilisées pour convertir les résultats: moment-vitesse de rotation, du viscosimètre à disques parallèles, dédié aux relations entre le taux de cisaillement circonférentiel et la contrainte de cisaillement, sont identifiées. La contrainte de cisaillement circonférentielle, obtenue par l'expression simplifiée, est comparée à la contrainte réelle. L'erreur introduite est quantifiée, pour 2 modèles rhéologiques standards et pour un ensemble de résultats de laboratoire. Dans les conditions normales d'utilisation du viscosimètre à disques plans, cette erreur s'avère être en dessous de la limite d'acceptabilité. Par contre, pour des fluides rhéofluidifiants fortement cisaillés, ou pour des fluides présentant un seuil d'écoulement en cisaillement, cette erreur peut devenir très importante. L'adéquation de l'approximation pour la contrainte de cisaillement circonférentielle, dans le cas d'une mesure de glissement le long d'un plan, est ensuite brièvement discutée.

Key words: Parallel-disk viscometer, rim shear stress, non-Newtonian viscosity, yield stress, ill-posed problem, wall slip

1 INTRODUCTION

The cone-and-plate and the parallel-disk viscometers are undoubtedly the two most commonly employed viscometers in rheological laboratories. They have many features in common but the parallel-disk geometry has a number of practical advantages over the cone-and-plate geometry. Chief among these are the relative ease with which the parallel disks can be set up and the continuously adjustable disk gap that gives the parallel disks the ability to cope with fluids with large suspended particles or droplets. In addition the adjustable disk gap also makes the parallel-disk viscometer a popular tool for investigating wall slip [1]. However, the parallel-disk geometry has a serious drawback. The shear rate $\dot{\gamma}$ experienced by the fluid under test varies significantly from the centre to the rim of the

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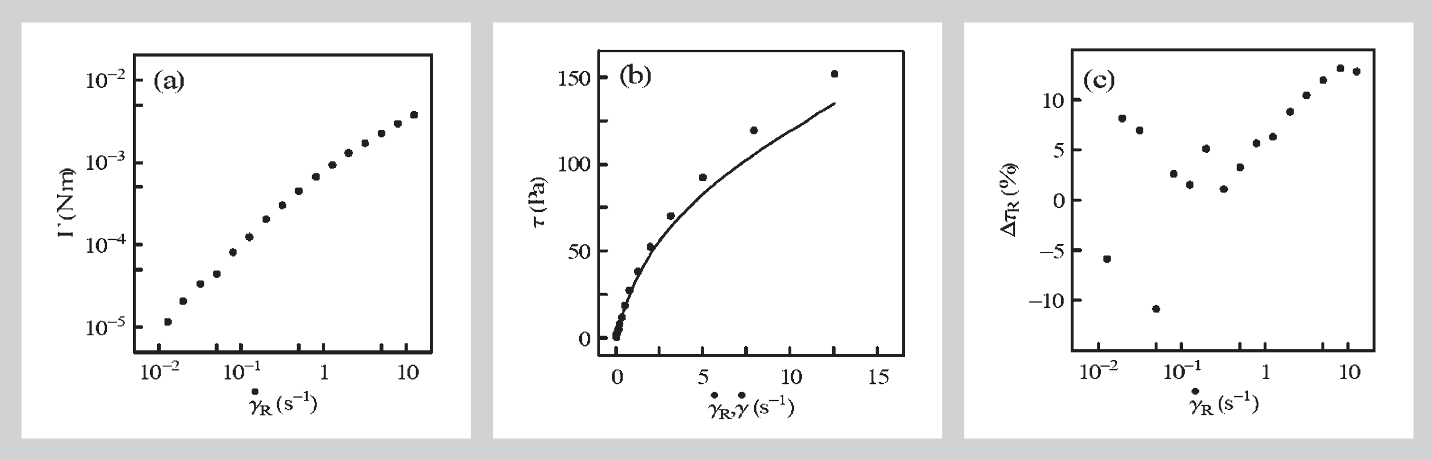

Figure 3: Data and results of hydroxypropyl methylcellulose solution.
a) (left) Torque versus rim shear rate data of Steffe [9].
b) (middle) Variation of τ_R^N and τ_R with shear rate. Points are τ_R^N based on Eq. 1b and the continuous curve is τ_R from Yeow et al. [8].
c) (right) Variation of $\Delta\tau_R$ with $\dot{\gamma}_R$ based on the τ_R^N and τ_R in Figure 3b.

A Hydroxypropyl Methylcellulose Solution

Steffe [9] reported the $(\Gamma, \dot{\gamma}_R)$ data of a 3% hydroxypropyl methylcellulose solution at 24.2°C. His data are reproduced in Figure 3a in log-log format to show up the data trend at very low rim shear rates. These data are for disk gap h = 0.7 mm and disk radius R = 25 mm.

Following the current common practice in processing parallel-disk data, Eq. 1b is ,used to convert these data, each point taken individually, into τ_R^N. The outcome is shown as discrete points in Figure 3b. Yeow et al. [8] applied Tikhonov regularization to convert the same set of data points, taking the entire set in one go, into a single shear rate-shear stress relationship $\tau(\dot{\gamma})$. For comparison, their result is shown as a continuous curve in Figure 3b. This plot is presented in linear format in order to reveal the error introduced by Eq. 1b at high shear rates. It should be stressed that the discrete points are based on the Newtonian equation while the curve is a model-independent description of the steady-shear property of the hydroxypropyl methylcellulose solution. The accuracy of the continuous curve has been checked by back calculations [8]. This curve can therefore be regarded as the "true" shear stress-shear rate relationship of the solution under test and because of the nature of Tikhonov regularization this curve is independent of any assumed rheological model. The difference between the discrete points and the continuous curve in Figure 3b is an indication of the error introduced by the simplified Newtonian-based Eq. 1b.

The difference between the rim shear stress given by the Newtonian-based expression and that reported by Yeow [8] is plotted as a percentage of the latter in Figure 3c. At high rim shear rates the Newtonian-based result has again over estimated the actual shear stress. The error there is between 5 to 10%. This is reasonably close to that expected for a power-law model with n ≈ 0.934 used by Steffe [9] to describe the shear rheology of this solution.

At low shear rates the $\Delta\tau_R$ does not exhibit any clear trend. The most noticeable feature there is that the Newtonian expression no longer consistently overestimates the actual rim shear stress. This random behavior can be traced to the experimental noise in the raw$(\Gamma, \dot{\gamma}_R)$ data at low rim shear rates which then shows up in the Newtonian-based τ_R^N. With the result from Tikhonov regularization the built-in regularization parameter has succeeded in damping out the noise in the data resulting in a relatively smooth $\tau(\dot{\gamma})$ curve. As mentioned above, Steffe [9] treated this hydroxypropyl methylcellulose solution as a power-law fluid and filtered out the noise by fitting a least-squares straight line through the data point. He obtained a set of $(\dot{\gamma}_R, \tau_R)$ data points that are in closer agreement with the Tikhonov regularization result than the comparison in Figure 3b. The error involved in this case is again around 10% and is therefore acceptable for many applications.

5 RIM SHEAR STRESS AND SLIP VELOCITY

From the three cases considered in this investigation, it can be seen that for most shear thinning fluids the Newtonian-based τ_R^N is likely to over estimate the true rim shear stress τ_R by 5 to 10%. As a general indicator of shear stress variation, the τ_R^N given by Eq. 1b is probably acceptable. However it is noted that the τ_R data, and in some cases τ_R^N, obtained for different disk gaps are used in the calculation of the wall slip function $v_{slip}(\tau_W)$ – the function relating slip velocity to wall shear stress τ_W based on the technique described by Yoshimura and Prud'homme [1]. In this technique the two rim shear stresses, for the same rim shear rate, from two difference disk gaps are compared against one another. Their difference is then used in the computation of $v_{slip}(\tau_W)$. When taking the difference between the two Newtonian-based rim shear stresses from two different gaps, especially when the stresses are not significantly different from one another, the effective error bar of the difference is likely

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to be very much larger than the 5 to 10% of the individual τ_R^N. For such calculations τ_R^N is clearly not acceptable as an approximation of the actual rim shear stress. In some of the published wall slip investigations it has not been made clear whether the $v_{slip}(\tau_W)$ was based on the difference of two actual rim shear stresses or the difference of two τ_R^N generated directly by the software that accompanies the parallel-disk viscometer used in the slip velocity investigation. Consequently such slip velocity functions will have to be treated with some caution. It is noted that in their original investigation of wall slip Yoshimura and Prud'homme [1] stated explicitly that they used the rim shear stresses obtained via the exact expression i.e. Eq. 3. They did not make any reference to the approximate Newtonian-based expression.

6 IMPROVED RIM SHEAR STRESS ESTIMATION

In a typical parallel-disk viscometer of today the raw torque-rotational data are often converted directly into a shear stress-shear rate relationship. Since the user is often not made aware of the approximations made, particularly that involving Eq. 1b, the purpose of the present investigation is to bring this to the attention of the users of what is clearly an increasingly more popular instrument. As the rheological model appropriate for the fluid under investigation is generally not known before hand or the fluid may not even be describable by any of the standard rheological models this rules out the direct evaluation of a model-based $\Delta\tau_R$ as an estimation of the error introduced by Eq. 1b. To make use of $\Delta\tau_R$ in any back-calculated correction for the rim shear stress would then involve a tedious iterative process. The reliability and the convergence of such a process have not been investigated. It is therefore not suggested that the kind of $\Delta\tau_R$ plots reported here be used in back calculations to obtain an improved estimate of the actual rim shear stress. A more fruitful approach would be to use the exact relationship given by Eq. 3 to convert the measured torque into true rim shear stress. This requires the evaluation of the derivative of a set of parallel-disk data which can be done by fitting an appropriate curve through the $(\Gamma, \dot{\gamma}_R)$ data and differentiating the fitted curve to obtain the required derivative on the RHS [9].

It should be borne in mind that differentiation of experimental data, however it is performed, is an ill-posed problem and requires careful consideration if noise amplification is to be kept under control. The procedure, based on Tikhonov regularization, reported recently by Yeow et al. [8] is an example of the specialized procedure used to deal with the ill-posed nature of the parallel-disk viscometry problem.

7 CONCLUSIONS

The simplified Newtonian-based expression for converting the measured torque into rim shear stress overestimates the actual rim shear stress, typically by around 5 to 10%. For highly shear thinning fluids and for fluid exhibiting yield stress this error can exceed 30%. Because of the large build up of error when calculating the difference between two rim shear stresses from two different disk gaps, the use of the simplified Newtonian-based expression in wall slip investigation is not recommended.

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