Free Surface Effects on Normal Stress Measurements in Cone and Plate Flow

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Abstract:
The effects of free surface shape on normal stress difference measurements in cone and plate flow are investigated. The analysis shows that the stress field is significantly altered by deviations of the free surface from an ideal (spherical) shape. For the cone and partitioned plate technique, it is shown how modest deviation from a spherical free surface shape can lead to errors of roughly 10% in the measured normal stress differences.

Zusammenfassung:
Der Einfluß der freien Oberfläche auf die Messung der ersten Normalspannungsdifferenz in der Kegel-Platte Geometrie wird diskutiert. Die Analyse zeigt, dass das Spannungsfeld erheblich durch Abweichungen von der Oberflächenidealform beeinflusst werden kann. Im Fall einer geteilten Kegel-Platte Anordnung können schon moderate Abweichung von der spherischen freien Oberfläche zu einem Fehler von 10% des Normalspannungsdifferenzwertes führen.

Résumé:
L' influence de la forme de la surface libre sur les mesures de différences de contraintes normales dans le cadre d' un écoulement cône-plan est étudiée. Cette analyse révèle que le champ des contraintes est fortement altéré lorsque la surface libre s'éloigne de sa forme idéale (sphérique). Concernant la technique de cône-plan partionné, il est montré comment de légères deviations par rapport à une surface libre sphérique peuvent conduire à des erreurs d' environ 10% sur la mesure des différences de contraintes principales.

Key words: normal stresses, free surface, Cone and Plate

1 INTRODUCTION

Cone-and-plate flow is widely used to study the rheological behavior of complex fluids. In most cone-and-plate rheometers, one of the fixtures, say the cone, is rotated and the torque and axial force are measured on the stationary plate, or vice-versa. The primary advantage of cone-and-plate flow is that the shear rate $\dot{\gamma}$ is approximately uniform within the fluid sample. Hence, unlike torsional flow between parallel disks, or pressure-driven flow in a capillary, the shear stress at a given shear rate $\sigma(\dot{\gamma})$ can be obtained from a single measurement, even for fluids displaying highly non-linear rheological behavior. In addition, the first normal stress difference $N_1(\dot{\gamma})$ can be obtained from a single axial force measurement. If the radial distribution of stress on the plate is measured, both $N_1(\dot{\gamma})$ and the second normal stress difference $N_2(\dot{\gamma})$ can be obtained.

There have been numerous analyses of cone and plate flow and the assumptions used that allow for the measurements described above to be made [1 - 4]. In this note, we examine the effects of the free surface between the test fluid and the surrounding ambient gas on measurements of N_1 and N_2 in a cone-and-plate rheometer. In particular, we focus on the cone and partitioned plate technique used by Meissner et al. [5] and more recently by Schweizer [6].

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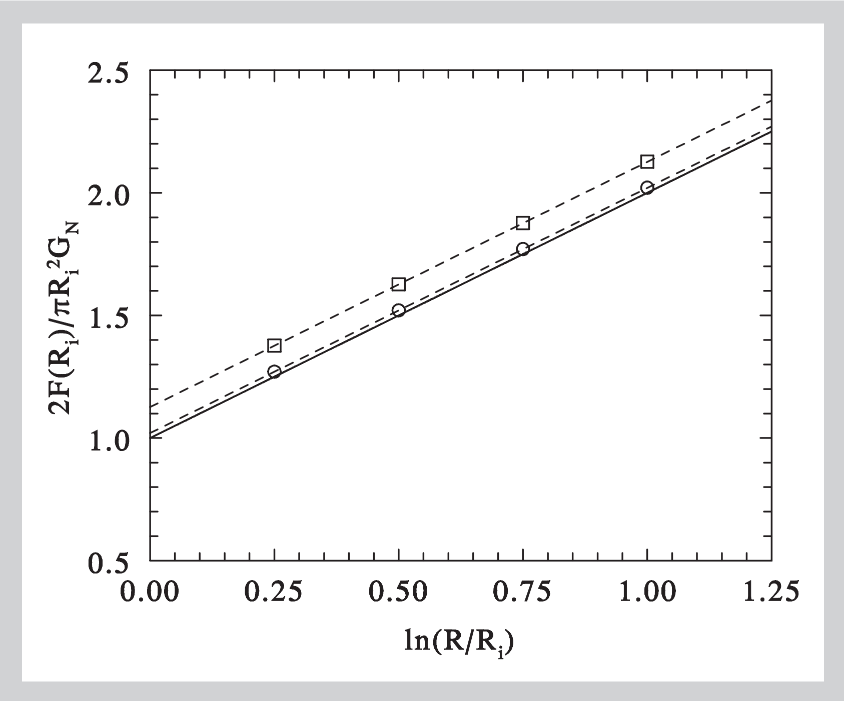

Figure 3: Normalized axial force on inner portion of plate with radius R_i as a function of normalized sample radius R. Solid line shows ideal result with Ca = 0 and a/R = 1. Symbols show 'measured' values for Ca = 0, α = 0.1 and different values of a/R: α(□), 2α(○).

For the case of a spherical surface ($a/R = 1$) and no interfacial tension ($\gamma = 0$), we recover the well-known result

$$-\pi_{\theta\theta}(r,\pi/2) = p_0 - N_2 - (N_1 + 2N_2)\ln\left(\frac{r}{R}\right) \tag{32}$$

which has been used to obtain N_1 and N_2 from measurements of the radial stress profile [7 - 9].

The net force (excluding the force from the surrounding gas) exerted on the inner portion of the plate by the fluid can be computed from

$$F(R_i) = \int_0^{2\pi}\int_0^{R_i} [\pi \cdot \mathbf{n}]_{\theta=\pi/2} \, r dr d\phi = 2\pi \int_0^{R_i} \pi_{\theta\theta}(r,\pi/2) r dr \delta_z \tag{33}$$

which, after substitution of Eq. 31, gives

$$\frac{2F(R_i)}{\pi R_i^2} = N_1 + 2(N_1 + 2N_2)\ln\left(\frac{r}{R}\right) + \frac{2\gamma}{R}\left(1 + \frac{R}{a}\right) - 2N_2 \frac{\alpha^2/4(R/a-1)^2}{1-\alpha^2/4(R/a-1)^2} \tag{34}$$

where $F = |\mathbf{F}|$. For the case of a spherical surface ($a/R = 1$) and no interfacial tension ($\gamma = 0$), we obtain from Eq. 34

$$\frac{2F(R_i)}{\pi R_i^2} = N_1 + 2(N_1 + 2N_2)\ln\left(\frac{R}{R_i}\right) \tag{35}$$

which is the expression used to obtain N_1 and N_2 from measurements of $F(R_i)$ as a function of the ratio R/R_i [5. 6]. Setting $R_i = R$ in Eq. 35 gives the well-known relation between N_1 and the total force on the plate:

$$\frac{2F(R)}{\pi R^2} = N_1 \tag{36}$$

3 RESULTS AND DISCUSSION

The analysis presented above shows that a non-spherical free surface affects the stress field in cone and plate flow. The reason for this can be seen in Eq. 24, which shows that both τ_{rr} and $\tau_{\theta\theta}$ are involved in the balance of the isotropic part of the stress tensor. This alteration of the stress field leads to additional terms in the measured forces used to obtain N_1 and N_2. As noted above, these effects imply the existence of an additional component of the extra stress tensor $\tau_{r\theta}$, which, in turn, would generate a secondary flow.

The main result of the analysis in the previous section is Eq. 34 which, when divided by a characteristic modulus for the fluid G_N, can be written as

$$\frac{2F(R_i)}{\pi R_i^2 G_N} = \frac{N_1}{G_N} + 2\frac{N_1}{G_N}(1+2\Psi)\ln\left(\frac{R}{R_i}\right) + Ca\left(1+\frac{R}{a}\right)\left(\frac{R_i}{R}\right) - 2\Psi\frac{N_1}{G_N}\frac{\alpha^2/4(R/a-1)^2}{1-\alpha^2/4(R/a-1)^2} \tag{37}$$

where $\Psi = -N_2/N_1$ and $Ca = 2\gamma/R_iG_N$. From Eq. 37, it is clear that interfacial tension affects the measured value of the intercept (N_1/G_N). If, as is often the case, the sample radius R is varied for a single value of R_i, interfacial tension would also affect the slope (Ψ). For polymer melts, Ca ~ 10^{-5}, so the errors introduced by interfacial tension would be negligible. However, for polymer solutions, Ca ~ 10^{-3} or larger, so interfacial tension could lead to errors for a bulged free surface. From this point on, we assume interfacial tension can be neglected.

To examine the effect of a non-spherical free surface, we set α = 1/10, N_1/G_N = 1 and Ψ = 1/4,

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which represent conditions for a typical experiment. Figure 3 shows 'measurements' of the axial force on the inner portion of the plate for two values of the ratio *a*/*R*, which controls the shape of the free surface. As shown in this figure, deviations from a spherical free surface (decreasing *a*/*R*) lead to errors in the measured intercept from which N_1/G_N is obtained. An error in N_1/G_N also leads to an error in the measured value of Ψ. For example, for $a/R = \alpha$ (squares in Figure 3), the error in N_1/G_N is approximately 13 %, which leads to an error of approximately 11 % in Ψ.

As noted earlier, the shape of the free surface (*a*/*R*) is not known and therefore, the example used above is only for illustrative purposes. It is also possible, in contrast to the example above, that the shape of the free surface (*a*/*R*) is a function of the sample size (R/R_i). This would directly affect the measured slope leading to an additional source of error in Ψ. It should also be noted that larger relative errors would be observed for larger values of α and Ψ.

CONCLUSIONS

The effects of free surface shape on normal stress difference measurements using the cone and partitioned plate technique have been investigated. The analysis presented here shows that modest deviations from a spherical free surface can lead to errors on the order of 10 % in measured values of first normal stress difference N_1 and ratio of normal stress differences N_2/N_1. These errors result from both interfacial tension and the modification of the normal stresses involved in the force balance at the free surface. This modification of the force balance also gives rise to an additional shear stress that would induce a secondary flow. Other possible sources of error, not considered here, are the dynamic nature of the free surface shape and sample flow in the gap between the inner and outer portions of the plate.

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