ZONE METHOD FOR REPRESENTING RELAXATION CHARACTERISTICS OF VISCOELASTIC MATERIALS

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ABSTRACT:
Exponential integral functions were fitted to relaxation data obtained from tensile and shear loading of an asphalt-sand mixture at different temperatures. This approach yields a better fit to the experimental data than the traditional Prony series and provides physical insight into essential characteristics of the relaxation processes that govern the asphalt-sand mixture. We expect that using this model beyond the time range covered by the experimental data would result in a significantly better representation of the material behavior than would extrapolation of the Prony series fit.

ZUSAMMENFASSUNG:
Exponentielle Integralfunktionen wurden Relaxationsdaten angepasst, die von Tests einer Asphalt-Sand-Mischung unter dem Einfluss verschiedener Temperaturen erhalten wurden. Diese Methode ermöglicht eine bessere Anpassung an die experimentellen Daten als die traditionelle Prony-Serie. Gleichzeitig bietet sie physikalische Einsicht in essentielle Eigen-schaften von Relaxationsprozessen, die das Verhalten von Asphalt-Sand-Mischungen bestimmen. Wir erwarten, dass man - verglichen mit der Extrapolation der Prony-Serie - bei Anwendung dieses Modells über den Zeitraum der experimentellen Daten hinaus eine signifikant bessere Repräsentation des Materialverhaltens erhält.

RÉSUMÉ:
Des fonctions intégrales exponentielles ont été ajustées à des données de relaxation obtenues lors de tests de tension et de cisaillement effectués sur un mélange de sable et d'asphalte à différentes températures. Cette approche produit un meilleur ajustement des données expérimentales que les séries de Prony traditionnelles, et apporte un traitement physique des caractéristiques essentielles des processus de relaxation qui gouvernent le mélange asphalte-sable. Nous espérons que l'utilisation de ce modèle au-delà de la gamme de temps accessible expérimentalement devrait aboutir à une représentation significativement meilleure du comportement du matériau que celle obtenue par extrapolation de l'ajustement effectué avec les séries de Prony.

KEY WORDS: Relaxation, viscoelasticity, asphalt, elastic after effect, Prony series, time constants

1 BACKGROUND & THEORY

A wide variety of materials display viscoelastic behavior. Viscoelastic behavior manifests itself as a time-dependent transient response to a step change in displacement or stress. The transient response resulting from the application of a load to different materials is often referred to as mechanical aftereffect or anelastic relaxation. The mechanical relaxation of materials has been studied and documented by many investigators [1]. A wealth of additional work describing analogous electrical relaxation effects in magnetic and dielectric materials also exists [2].

When modeling the time-dependent beavior of materials, it is critical to have a good mathematical description of the material response as a function of time and temperature. In several research areas, the formalism shown in Equation 1 is used and is referred to by several different names. In the study of the mechanical response of polymers, it is typically called the Prony series. The Prony series is used to describe the time-dependent behavior, and the temperature superposition principle (or WLF theory) [3] is used to describe the temperature dependence of that behavior. The Prony series is given by:

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Using this as a basis, we see that a distribution of activation energies, $p(E)$, will result in a distribution of time constants $p(\tau)$. For our purposes, it is not essential to consider the microscopic details of the origin of the distribution of activation energies, but rather to establish that the distribution of time constants already cited (Eqs. 7 and 8) corresponds to a sensible distribution of activation energies. This is shown below.

Starting with Eq. A-1 it is desired to find the probability density function associated with the random variable E, for the case where the PDF for τ is expressed by Eq. 7 and 8. Because the PDF of τ takes a non-zero, non-negative value over the range $\tau_1 < \tau < \tau_2$, the PDF of E is defined for the range $E_1 < E < E_2$, where E_1 corresponds to τ_1 and E_2 to τ_2. Notice that if $E_1 = 0$, $\tau_1 = \tau_0$, a finite positive number.

We now make use of the theory of functions of a random variable. The monotonic behavior of the transformation in Eq. A-1 allows us [11] to compute the PDF for E directly from the PDF for τ. That relationship is expressed by

$$p(E) = p(\tau)\left|\frac{d\tau}{dE}\right| \qquad \text{(A-2)}$$

In mathematical terms, $d\tau/dE$ is the Jacobian of the transformation. We now compute $p(E)$. Performing the indicated differentiation and substituting the result into Eq. A-2 yields:

$$p(E) = \frac{1}{\tau_o e^{E/RT} \ln\frac{\tau_2}{\tau_1}} \frac{\tau_o}{RT} e^{E/RT} \qquad \text{(A-3)}$$

The first factor in Eq. A-3 is $p(\tau)$ and the second factor is $|d\tau/dE|$. Equation A-3 may be simplified to read:

$$p(E) = \frac{1}{RT \ln\frac{\tau_2}{\tau_1}} \qquad \text{(A-4)}$$

Because Eq. A-4 does not show the dependence of $p(E)$ on E explicitly, we must express Eq. A-4 in terms of E. This is accomplished by employing the relationships below, which follow by inspection of the previous development.

$$\tau_1 = \tau_o e^{E_1/RT}$$
$$\tau_2 = \tau_o e^{E_2/RT}$$
$$\frac{\tau_2}{\tau_1} = e^{(E_2 - E_1)/RT}$$
$$\ln\left(\frac{\tau_2}{\tau_1}\right) = \frac{1}{RT}(E_2 - E_1)$$

Substituting the last expression above in Eqn. A-4, we find:

$$p(E) = \begin{Bmatrix} \dfrac{1}{E_2 - E_1} & E_1 < E < E_2 \\ 0 & \text{elsewhere} \end{Bmatrix} \qquad \text{(A-5)}$$

This is the desired result, *i.e.*, the PDF associated with the activation energy E is constant (with a value of $1/(E_2 - E_1)$) over the energy range $E_1 < E < E_2$, and zero elsewhere.

In essence, this describes a situation in which higher activation energies correspond to relaxation processes with longer time constants. It is very likely that $p(E)$ does not exhibit discontinuities, but is characterized by a relatively flat (slightly rounded) amplitude over the activation energy range $E_1 < E < E_2$ and rapidly decays to zero outside that range. Little attempt has been made to refine the PDF of τ (and E) mathematically, primarily because the exponential integral model used to date allows a curve fit to data from viscoelastic relaxation, $\psi(t)$, that is sufficiently close to fulfill the demands of most applications.

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