Is the Choice of Flow Curve Fitting Equation Crucial for the Estimation of Pumping Characteristics?

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Abstract:
The answer to this provocative question is "no"! This is demonstrated by experiment and analysis for two very different materials – a highly concentrated emulsion and an 8%v/v Kaolin clay suspension. The flow curves of both materials clearly showed a low shear Newtonian asymptote and a pseudoplastic domain. The difference in the accuracy of the fitting equations relates mainly to the low shear rate domain. While the Cross equation is adequate over the full flow curve, the power law and the Herschel-Bulkley equations are clearly inadequate for the low shear rate range. These equations as well as the direct numerical method (using the Rabinowitsch-Weissenberg integral) were used for the calculation of the laminar pipe flow transport characteristics and the results were compared with experimental pipe flow data. It was shown that in all cases the maximum error did not exceed 5%, which is quite acceptable for engineering design, indicating that the choice of the flow curve fitting equation was unimportant.

Zusammenfassung:
Die Antwort auf diese provokative Frage ist „nein"! Dies wird an Hand von Experimenten und Analyse von zwei sehr unterschiedlichen Materialien, einer hoch konzentrierte Emulsion und einer Kaolin-Tonerde-Suspension gezeigt. Die Fliesskurven beider Materialien zeigen klar ein asymptotisches Verhalten bei kleinen Scherraten und Scherverdünnung bei höheren Scherraten. Fitfunktionen zeigen hauptsächlich Unterschiede im Bereich kleiner Scherraten: Die Cross-Gleichung ist für den gesamten Scherratenbereich hinreichend, während der Potenzansatz und die Herschel-Bulkley-Gleichungen für den Bereich kleiner Scherraten eindeutig unzulänglich sind. Sowohl diese Gleichungen als auch die direkte Berechnung (unter Benutzung des Rabinowitsch-Weissenberg-Integrals) wurden zur Bestimmung der laminaren Fliess- und Transporteigenschaften in einem Rohr benutzt und die Ergebnisse wurden mit experimentellen Daten von Rohrströmungen verglichen. Es wurde gezeigt, dass der maximale Fehler in allen untersuchten Fällen nicht über 5% lag, was für ingenieurtechnisches Design völlig akzeptabel ist und zeigt, dass die Wahl der Gleichung für die Beschreibung der Fliesskurve nicht von Wichtigkeit ist.

Résumé:
La réponse à cette question provoquante est «non». Ceci est démontré par des expériences et leur analyse obtenues pour deux matériaux très différents – une émulsion hautement concentrée et une suspension d'argile de Kaolin à 8% en volume. Les courbes d'écoulement des deux matériaux montrent clairement une asymptote Newtonienne à faible cisaillement, et un domaine pseudo-plastique. La différence de précision entre les équations d'ajustement des données est reliée principalement au domaine de basse vitesse de cisaillement. Tandis que l'équation de Cross est adéquate pour décrire la courbe d'écoulement en entier, les équations de loi de puissance et de Herschel-Bulkley sont clairement inadéquates pour le régime de faible cisaillement. Ces équations, de même que la méthode numérique directe (utilisant l'intégrale Rabinowitsch-Weissenberg) ont été utilisées pour calculer les caractéristiques d'un écoulement laminaire dans un transport par tube, et les résultats ont été comparés avec les données expérimentales d'écoulement en tube. Il a été montré que dans tous les cas l'erreur maximale n'excédait pas 5%, ce qui est assez acceptable pour une conception d'ingénierie, ce qui indique que le choix de l'équation pour ajuster la courbe d'écoulement n'était pas important.

Key words: emulsion, Kaolin, flow curves, Cross equation, power law, Herschel-Bulkley equation, Rabinowitsch-Weissenberg equation, laminar pipe flow

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Experiments carried out in this work related to two different materials - a highly concentrated emulsion and a 8% Kaolin suspension. The full flow curves of both materials consist showed a clearly defined low shear rate Newtonian asymptote and a pseudoplastic domain. The difference in the "goodness of fit" of the fitting equations relates principally to the low shear rate domain. The Cross-type equation is adequate for the full flow curve, while the power law and the Herschel-Bulkley equation are inadequate for describing the low shear rate behaviour because the observed flow curves follow a Newtonian plateau. These three equations were used for the prediction of pipe flow behaviour. Additionally, the Rabinowitsch-Weissenberg integral was evaluated numerically, without any analytical approximation of the flow.

Our principal finding is that, in all cases, the calculated $Q(\Delta P)$ dependencies were very close to the experimental pipe flow data obtained under typical industrial flow conditions. This finding stands - regardless of the obvious inadequacy of some of the flow curve approximations in the low shear rate domain. The maximum prediction error in all cases did not exceed 5%, which is acceptable for engineering design. Indeed maximal variations in real pipeline experiments reached ±6% which is larger than the error in model predictions.

This finding is explained by the fact that the governing factor for predicting transport characteristics of pipelines in real engineering conditions is flow with high shear rates while low shear rate domain gives only very limited input in integral flow rate. Different models are very close to each other in the high shear rate domain, which determines flow rate in practically important velocity range.

This same conclusion applies to the pipe flow of polymer melts. Indeed, the flow curve of a melt can be presented either by the Cross-type equation over the full shear rate range or by the power law, provided that the low shear rate Newtonian behavior is excluded. Predictions of the $Q(\Delta P)$ dependencies, based on these equations, will appear practically identical over the range of industrial interest.

The results of this study should not be regarded as a denial of the interest and importance of the low shear rate domain of the flow curves of industrially relevant materials and the discussion [13 - 17] concerning the existence and possible methods of determining the yield stress has an important fundamental basis. However this interest lies mainly outside the field of engineering applications and relates rather to the correlation of viscous properties of a material in this shear rate range with chemical composition and the structural peculiarities of rheologically complex materials. While the choice of rheological model might be not crucial for high-speed technological processes, such as transportation and processing, there are such situations where low speeds of flow are important, such as tailings deposition, product shelf life, and many others. In the latter case the description of the rheological behavior in the low shear rate domain remains crucial. Besides, rheological properties of a material (in particular polymeric materials) is especially sensitive for the details of molecular structure and preferably can be used in this case [18].

REFERENCES

[1] Tanner R: Engineering rheology, 2nd Ed., Oxford Univ. Press 2000.

[2] Slatter P T (1999); The role of rheology in the pipelining of mineral slurries, Min. Pro. Ext. Met. Rev., Vol20, pp 281-300.

[3] Slatter P T (2002); Non-Newtonian Laminar Pipe Flow – A Place In The Sun At Last!; Invited Keynote Address - 11th International Conference on Transport and Sedimentation of Solid Particles - Ghent, September, 2002. ISBN 83-87866-34-2, pp33-40.

[4] Slatter P T and Wasp E J (2002); Yield stress - How low can you go?; 11th International Conference on Transport and Sedimentation of Solid Particles - Ghent, September, 2002. ISBN 83-87866-34-2, pp173-182.

[5] M.M. Cross: Rheology of non-Newtonian fluids: a new flow equation for pseudo-plastic systems, J. Colloid Sci. 20 (1965) 417-437.

[6] I. Masalova, A.Ya. Malkin, et al: The rheological characterization and pipeline flow of high concentration water-in-oil emulsions, J. Non-Newton. Fluid Mech. 112 (2003) 101-114.

[7] G.W. Govier and K. Aziz: The flow of complex mixtures in pipe, van Nostrad Reinhold Co. 1972.

[8] A.Ya. Malkin, I. Masalova: Effect of droplet size on the rheological properties of highly concentrated w/o emulsions, Rheol. Acta, accepted.

[9] S.C. Nicol, R.J. Hunter: Some rheological and electrokinetic properties of kaolinite suspensions, Aus, J. Chem. 23 (1970) 2177- 2181.

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http://www.appliedrheology.org

[10] J.W. Goodwin: Rheological studies of on the dispersions of kaolinite suspensions, Trans. J. Brit. Ceramic Soc. 70 (1971) 65-72.
[11] B. Rand, I.E. Melton: Particle interaction in aqueous kaolinite suspensions. I. Effect of an electrolyte upon the mode of interaction in homoionic sodium kaolinite suspensions, J. Colloid Interface Sci. 60 (1977) 308-320.
[12] S. Jogan, C.F. Zukoski: Rheology of dense suspensions of plate like particles, J. Rheol. 40 (1996) 1211-1232.
[13] H.A. Barnes and K. Walters: The yield stress myth? Rheol. Acta 24 (1985) 323-326.
[14] Q.D. Nguyen, D.V. Boger: Measuring the flow properties of yield stress liquids, Ann. Rev. Fluid Mech. 24 (1992) 47-88.
[15] N. Pashias, D.V. Boger, et al: A fifty cent rheometer for yield stress measurements, J. Rheol. 40 (1996) 1179-1190.
[16] G.R. Burgos, A.N. Alexandrou, et al: On the determination of yield surfaces in Hershel-Bulkley fluids, J. Rheol. 43 (1999) 463-484.
[17] H.A. Barnes: The yield stress – a review or "$\pi\alpha\nu\tau\alpha$ $\rho\epsilon\iota$" – everything flows? J. Non-Newton. Fluid. Mech. 81 (1999) 133-178.
[18] G.V. Vinogradov, A. Ya. Malkin, Rheology of polymers, Springer Verlag, 1980.

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