Phenomenological Modelling of Non-Monotonous Shear Viscosity Functions

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Abstract:
The aim of this paper is to present a new phenomenological rheological model suitable for the description of a wide class of viscoelastic fluids. Classical phenomenological models predict the relation shear viscosity vs. shear rate (or shear stress) for shear-thinning (or thickening) materials exhibiting smooth monotonous passage from the first - upper (lower) - Newtonian plateau to the second - lower (upper) - one. However, present state of non-Newtonian materials used in practice (ranging from aqueous surfactant solutions, bituminous materials, associative polymers, polymer thickeners, lacquers and gels, to some special disperse systems, etc.) evokes the need to describe this - for many materials non-monotonous - relation in the corresponding way, i.e. through the sufficiently simple phenomenological model with a moderate number of parameters. A six-parameter model enabling description of not only monotonous but also non-monotonous course of shear viscosity function against shear rate (stress) is proposed including physical characterisation of the parameters. This model describes not only extreme points (maximum or minimum) but also a possible appearance of intermediate Newtonian plateau or its indication. The meaning and influence of the individual six parameters is documented on the experimental data published in the literature. There is a good agreement of the model proposed with many different experimental data representing different rheological behaviour. The applicability of this model for a wide class of viscoelastic materials is its principal advantage over the hitherto published phenomenological models.

Zusammenfassung:
In diesem Artikel wird ein phänomenologisches rheologisches Modell vorgestellt, welches zur Beschreibung der Fliesseigenschaften einer grossen Anzahl viskoelastischer Fluide geeignet ist. Klassische phänomenologische Modelle beschreiben die Abhängigkeit der Scherviskosität als Funktion der Scherrate (oder Schubspanung) für scherverdünnende (oder scherverdickende) Materialien, welche einen stetigen, monotonen Übergang von einem ersten zu einem zweiten newtonschen Plateau besitzen. Die meisten verwendeten nicht-newtonschen Materialien (welche von wässrigen Tensidlösungen, bituninöser Materialien, assoziierender Polymere, Polymereindicker, Lacken und Gelen, bis zu speziellen dispersen Systemen reichen), verlangen eine Beschreibung des für viele Materialien nicht-monotonen Zusammenhanges in einer angemessenen Art und Weise, d. h. durch ein hinreichend einfaches phänomenologisches Modell mit einer angemessenen Anzahl von Parametern. Ein sechs-Parameter Modell samt physikalischer Interpretation der Parameter wird vorgeschlagen, welches sowohl einen monotonen als auch einen nicht-monotonen Verlauf der Scherviskosität als Funktion der Scherrate (Schubspannung) beschreiben kann. Dieses Modell beschreibt nicht nur Extremalpunkte (Maxima und Minima), sondern auch ein mögliches Auftreten von dazwischenliegenden Newtonschen Plateaus oder deren Andeutung. Die Bedeutung und der Einfluss der einzelnen sechs Parameter wird an Hand publizierter experimenteller Daten dokumentiert. Es besteht eine gute Übereinstimmung des vorgeschlagenen Modells mit verschiedenen experimentellen Befunden, die ein unterschiedliches rheologisches Verhalten darstellen. Die Anwendbarkeit dieses Modelles auf eine grosse Klasse viskoelastischer Materialien ist der Hauptvorteil im Vergleich zu den bis heute veröffentlichten phänomenologischen Modellen.

Résumé:
Le but de cet article est de présenter un nouveau modèle rhéologique phénoménologique, convenable pour la description d'une grande classe de fluides viscoélastiques. Les modèles phénoménologiques classiques prédisent une relation entre la viscosité de cisaillement et la vitesse de cisaillement (ou la contrainte de cisaillement) pour des matériaux rhéo-amincissant (ou épaississant) qui présentent un passage continu et monotone entre le premier – le plus elevé (bas) - plateau Newtonien et le second plateau – le plus bas (élevé). Cependant, l'état présent des matériaux non-Newtoniens utilisés dans la pratique (qui vont des solutions aqueuses de surfactants, des matériaux bitumeux, polymères associatifs, polymères épaississant, laques et gels, jusqu'à certains systèmes dispersés spéciaux) évoque le besoin – pour de nombreux matériaux non monotones -de décrire cette relation de la manière suivante, c-à-d, à travers un modèle phénoménologique suffisamment simple avec un nombre modéré de paramètres. Un modèle à six paramètres permettant la description de non seulement l'évolution monotone mais aussi non monotone de la fonction de viscosité de cisaillement en fonction de la vitesse de cisaillement (contrainte de cisaillement) est proposé en incluant une caractérisation physique des paramètres.

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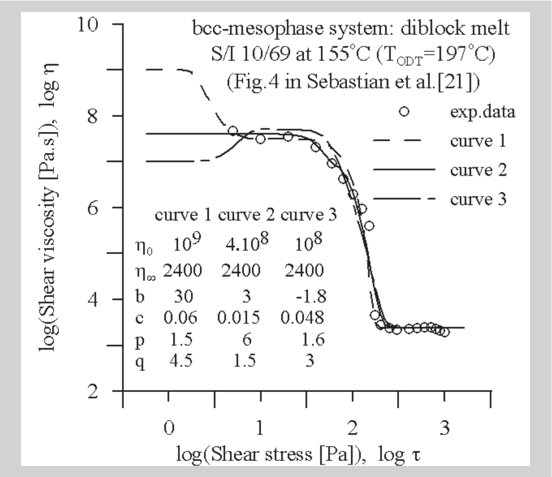

Figure 7: Non-uniqueness in a determination of η_o.

where

$$f(\tau;c,p,q)=sign\left(\log(c\tau)^{p}\right)\left|\log(c\tau)^{p}\right|^{q} \tag{5b}$$

The range of the experimental data strongly influences the choice of the values η_o, η_∞. Figure 7 illustrates that even the same model provides various sets of parameters with equivalently satisfactory approximation of the experimental data (in Fig. 7 the sets of parameters are not fully optimised in order to distinguish the curves visually). The proper value of η_o can be questioned in the way similar to that for the determination of the yield stress τ_o (Barnes and Walters [22], Barnes [1]).

A number of parameters in the model (Eqs. 4a, b, 5a, b) exceeds the one used in the classical models (two to five), but these models are restricted solely to the description of monotonous behaviour of flow curves. A six-parameter non-monotonous model in Evans [11]

$$\frac{\eta-\eta_\infty}{\eta_o-\eta_\infty}=\frac{1+k_2\dot{\gamma}^{n_2}}{1+k_1\dot{\gamma}^{n_1}} \tag{6}$$

proceeds from a functional form introduced by the classical models, and in spite of its possibility to model local maximum or minimum it fails in modelling more abrupt changes in behaviour of flow curves (such as in Figs. 4, 7) or respecting an intermediate plateau or its indication (as in Fig. 6). In respect to these facts the number of parameters – six – in the model proposed seems to be adequate.

4 CONCLUSION

The proposed six-parameter model enables to model non-monotonous viscosity functions in dependence on shear rate or shear stress. The participation of the individual parameters in the functional behaviour of a viscosity function is presented as well as the procedure how to consecutively estimate these parameters. No parameter in the model exhibits unstable behaviour. The model is applied to the experimental data already published in the literature. This documents, and this is the principal advantage over the hitherto published rheological models, that the model proposed is in agreement with many different experimental results representing many different rheological behaviour.

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