Simulation of Lubricated Squeezing Flow of a Herschel-Bulkley Fluid Under Constant Force

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Abstract:
Lubricated squeezing flow (LSF) of a Herschel-Bulkley fluid between parallel disks under constant force was theoretically analyzed. An analytical expression for the fluid thickness as a function of time was obtained in terms of a hypergeometric function. The fluid thickness profiles in LSF were simulated for a range of each of the model parameters (n, K, τ_o). The solution obtained in this study reduces to the corresponding analytical equations previously derived for LSF of Newtonian and power-law fluids. The simulations for Herschel-Bulkley fluid were compared with the response of Newtonian and power-law fluids. The dependence of the limiting fluid thickness (i.e. $H(t)/H_O$ at 180 s) on model parameters is presented.

Zusammenfassung:
Die Gleit-Quetschströmung (Lubricated Squeezing Flow) einer Herschel-Bulkley Flüssigkeit wird theoretisch für den Fall von sich parallel bewegten Platten unter konstanter Kraft untersucht. Man erhält einen analytischen Ausdruck für die Dicke der Flüssigkeit als Funktion der Zeit in Form einer hypergeometrischen Funktion. Die Profile der Flüssigkeitsdicke der Gleit-Quetschströmung wurden für verschiedene Werte für jeden der Parameter (n, K, τ_o) simuliert. Die in dieser Arbeit erhaltene Lösung reduziert sich auf die entsprechende analytische Gleichung, die für die Gleit-Quetschströmung von newtonschen und Power-Law Flüssigkeiten hergeleitet wurde. Die Simulationen von Herschel-Bulkley Flüssigkeiten wurden mit der Antwort von newtonschen und Power-Law Flüssigkeiten verglichen. Die Abhängigkeit der limitierenden Flüssigkeitsdicke (i.e. $H(t)/H_o$ bei 180 s) von den Modelparametern wird gezeigt.

Résumé:
L'écoulement de pression lubrifié (LSF) d'un fluide Herschel-Bulkley entre des disques parallèles et en régime de force constante, a été analysé théoriquement. Une expression analytique pour l'épaisseur de fluide en fonction du temps a été obtenue sous la forme d'une fonction hypergéometrique. Les profils d'épaisseur de fluide en LSF ont été simulés en faisant varier les paramètres du modèle (n, K, τ_o). La solution obtenue dans cette étude se réduit aux équations analytiques correspondantes précédemment obtenues pour le LSF de fluides Newtoniens ou à loi de puissance. Les simulations pour un fluide Herschel-Bulkley ont été comparées avec la réponse de fluides Newtoniens et à loi de puissance. La dépendance de l'épaisseur limite du fluide (i.e. $H(t)/H_O$ à 180s) en fonction des paramètres du modèle, est présentée.

Key words: Squeezing flow, extensional yield stress, limiting fluid thickness, coating thickness, spreadable foods

1 INTRODUCTION

Lubricated squeezing flow (LSF) is a rheological technique that involves compression of a fluid between parallel plates under the perfect slip conditions at the plate-fluid interfaces [1]. LSF is commonly used for measuring rheological properties of semi-solid foods such as peanut butter, cream cheese, melted cheese, tomato paste, butter, ketchup, mustard, mayonnaise, and yogurt [2-14]. The major advantage of the LSF technique is that food rheologists can avoid or minimize problems encountered during rheological measurements of such products with conventional rotational viscometers (e.g., damage incurred during sample loading; possibility of slip during measurement) [13].

Analytical and numerical solutions for LSF of various fluids allow determination of elongational properties from experimental data. For instance, elongational viscosity of a Newtonian fluid can be obtained from the thickness-time data using the following equation derived for LSF under constant force and constant volume conditions:

$$\frac{1}{H(t)} = \frac{1}{H_o} + \frac{W}{3\,\mu\,\Lambda}\,t \tag{1}$$

The corresponding equation for a power-law fluid [15] can be expressed as follows:

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obtaining desired properties for such foods. For instance, consumers often expect low-fat products to perform as good as the original versions. Using data from Ak and Gunasekaran we calculated normalized thickness values for regular and fat-free mayonnaises as 0.72 versus 0.59, 0.66 versus 0.51, and 0.63 versus 0.49, respectively, at 10, 50 and 180 s under 0.49 N load [14]. It remains, however, to be determined whether such differences are perceptible and important for sensory quality of mayonnaise.

4 CONCLUSIONS

The lubricated squeezing flow (LSF) of Herschel-Bulkley fluid between parallel disks has been analyzed under constant-force and constant-volume configuration. An analytical expression linking fluid thickness and time was obtained in terms of a hypergeometric function. The decrease in normalized thickness of the fluid was proportional to the increase in applied squeezing force. For a given yield stress the fluid thickness profile could be greatly modified by varying the consistency index. A greater decrease in the fluid thickness was predicted for the flow behavior index approaching to Newtonian value. As expected, the higher the value of (extensional or shear) yield stress the lower was the amount of squeezing before attaining the limiting thickness. The limiting thickness graphs as a function of rheological parameters are expected to be useful in understanding behavior of materials with yield stress (e.g., spreadable foods).

NOTATIONS

F(t)	Momentary squeezing force (N)
H_L	Apparent limiting thickness (m)
H_o	Initial fluid thickness (m)
H(t)	Momentary fluid thickness (m)
K	Consistency index (Pa s^n)
n	Flow behavior index (-)
R_o	Radius of squeezing disk (m)
R(t)	Momentary radius of fluid (m)
t	Time (s)
W	Applied constant force (N)
μ	Viscosity (Pa s)
σ_i	Applied initial stress (Pa)
σ_o	Extensional yield stress (Pa)
τ_o	Shear yield stress (Pa)
Λ	Volume of fluid (m^3)

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[1] There are some typographical errors in Yang's article (F. Yang, 1999, personal communication).

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