Fluctuations in laminar flow

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Hydrodynamics

Balance of momentum:

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) = -\boldsymbol{\nabla}[(\rho \mathbf{v})\mathbf{v}] \boxed{+\boldsymbol{\nabla}\Pi - \boldsymbol{\nabla} p + \mathbf{f}_{\mathrm{V}}}$$

Entropy production:

$$\dot{\mathcal{S}} = \frac{1}{T}\,\Pi : (\boldsymbol{\nabla}\mathbf{v})$$

Linear phenomenological laws (incompressible flow):

$$\Pi_{ij} = \sum_{kl} \eta(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\frac{\partial v_k}{\partial x_l}$$

Fluctuating hydrodynamics I

- Dissipation is due to molecular collisions (interaction)
- To reflect the random nature of these collisions, in fluctuating hydrodynamics dissipative fluxes are supplemented with stochastic contributions. For incompressible flow:

$$\Pi_{ij} = \eta(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\frac{\partial v_k}{\partial x_l} + \delta\Pi_{ij}$$
$$= \eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \delta\Pi_{ij}$$

- Stochastic properties: $\langle \delta\Pi_{ij} \rangle = 0$, and fluctuation-dissipation theorem:

$$\langle \delta\Pi_{ij}(\mathbf{r}, t)\ \delta\Pi_{kl}(\mathbf{r}', t') \rangle = 2k_{\mathrm{B}}T \boxed{\ \eta(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\ }\ \delta(\mathbf{r} - \mathbf{r}')\ \delta(t - t')$$

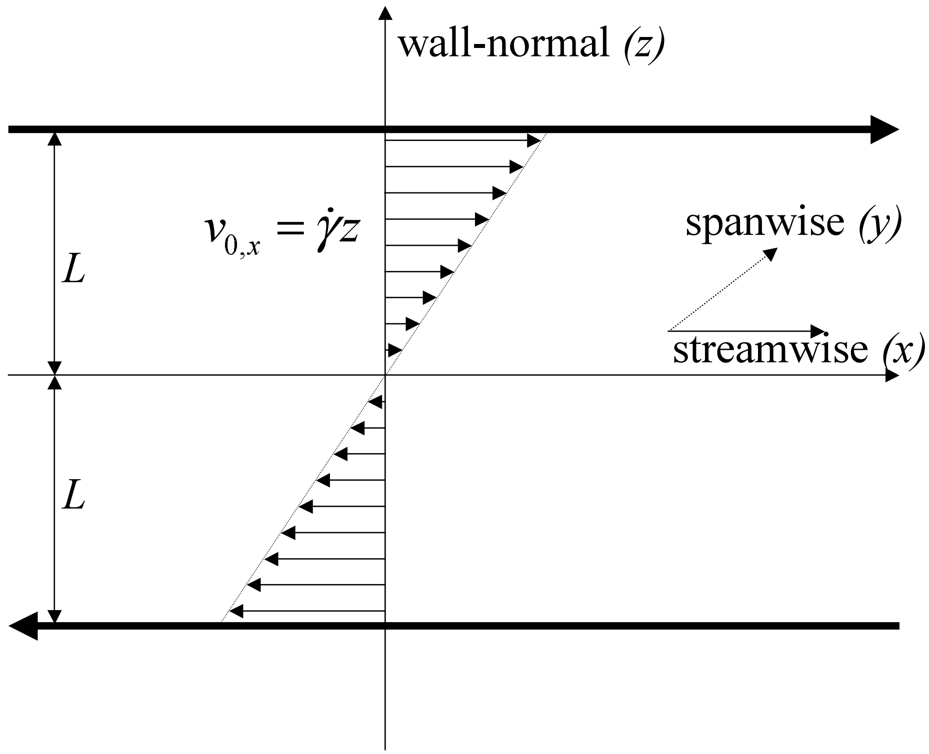
Fluctuating hydrodynamics II

- Substituting into momentum balance, we obtain an stochastic Navier-Stokes equation, with a $\delta\Pi$ forcing term. For incompressible flow and no volume forces:

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla}) \mathbf{v} \right] = -\boldsymbol{\nabla} p + \eta \nabla^2 \mathbf{v} + \boldsymbol{\nabla} \delta\Pi$$

Plane Couette flow I

Solution of the deterministic equation



Plane Couette flow II

- Studying velocity fluctuations around the plane Couette flow, in the usual way, we obtain stochastic Orr-Sommerfeld and Squire equations for wall normal velocity δv_z and vorticity $\delta\omega_z$ fluctuations:

$$\partial_t(\nabla^2\delta v_z) + z\partial_x(\nabla^2\delta v_z) - \frac{1}{\mathrm{Re}}\nabla^4\delta v_z = \{\nabla\times\nabla\times\nabla\delta\Pi\}_z$$

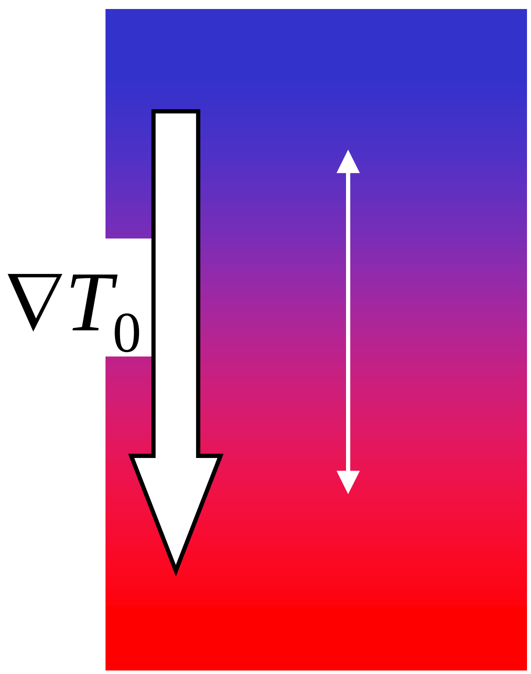
$$\partial_t(\delta\omega_z) + z\partial_x(\delta\omega_z) - \boxed{\partial_y\delta v_z} - \frac{1}{\mathrm{Re}}\nabla^2\delta\omega_z = \{\nabla\times\nabla\delta\Pi\}_z$$

- Problem: deduce the fluctuations of fields $\langle\delta v_z(\mathbf{r},t)\delta v_z(\mathbf{r}',t)\rangle$ and $\langle\delta\omega_z(\mathbf{r},t)\delta\omega_z(\mathbf{r}',t)\rangle$ from the stochastic properties of the thermal noise.

- Non-equilibrium enhancement due to mode-coupling

Nonequilibrium enhancement of thermal noise

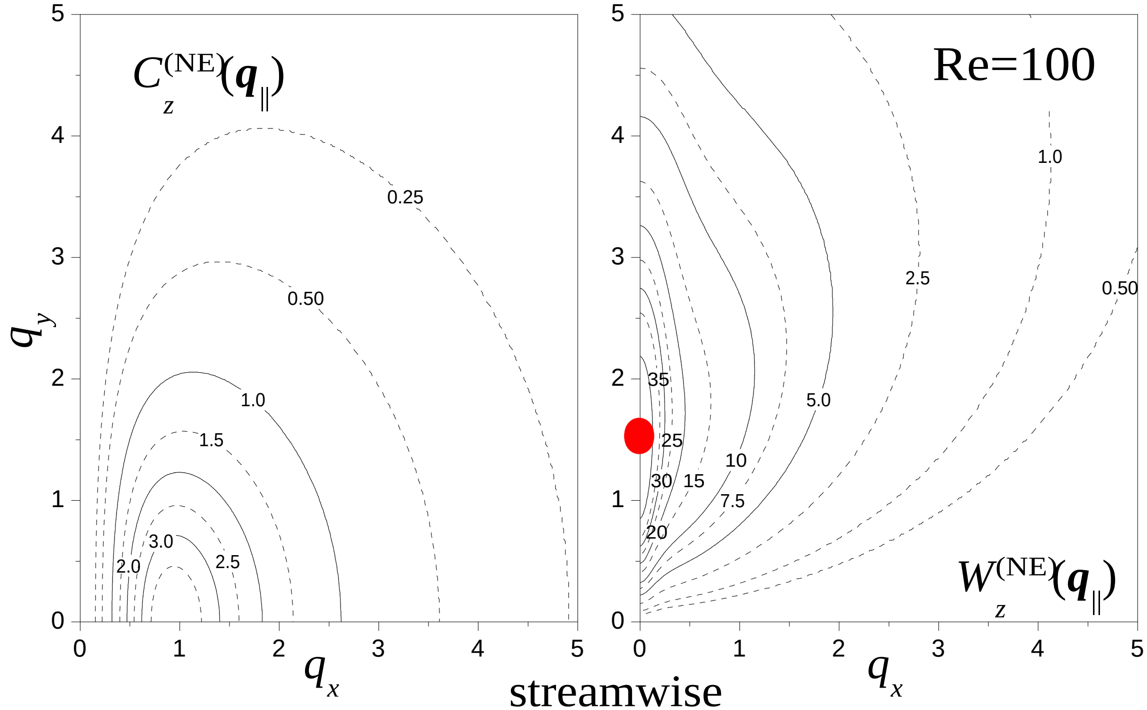
In nonequilibrium systems thermal noise is generically amplified due to mode coupling. Illustration with a temperature gradient.



- Fluctuations in δv_z "mix" regions with different (local) temperature.
- Advective term $\nabla T_0 \, \delta v_z$ in hydrodynamic equations.
- Local version of FDT.
- Problem: What are the fluctuations maximally enhanced?

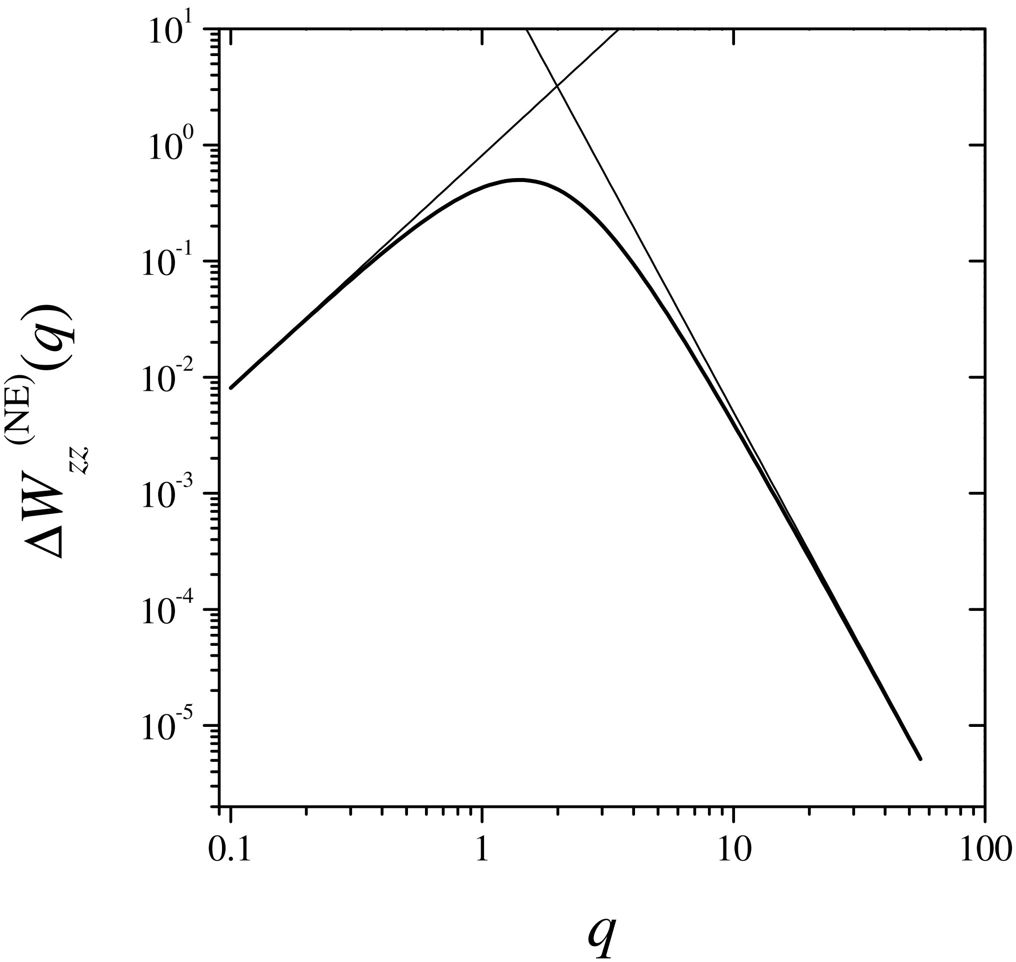
Amplification of thermal fluctuations in plane Couette

Most important effect: vorticity fluctuations enhancement due to coupling with Orr-Sommerfeld.



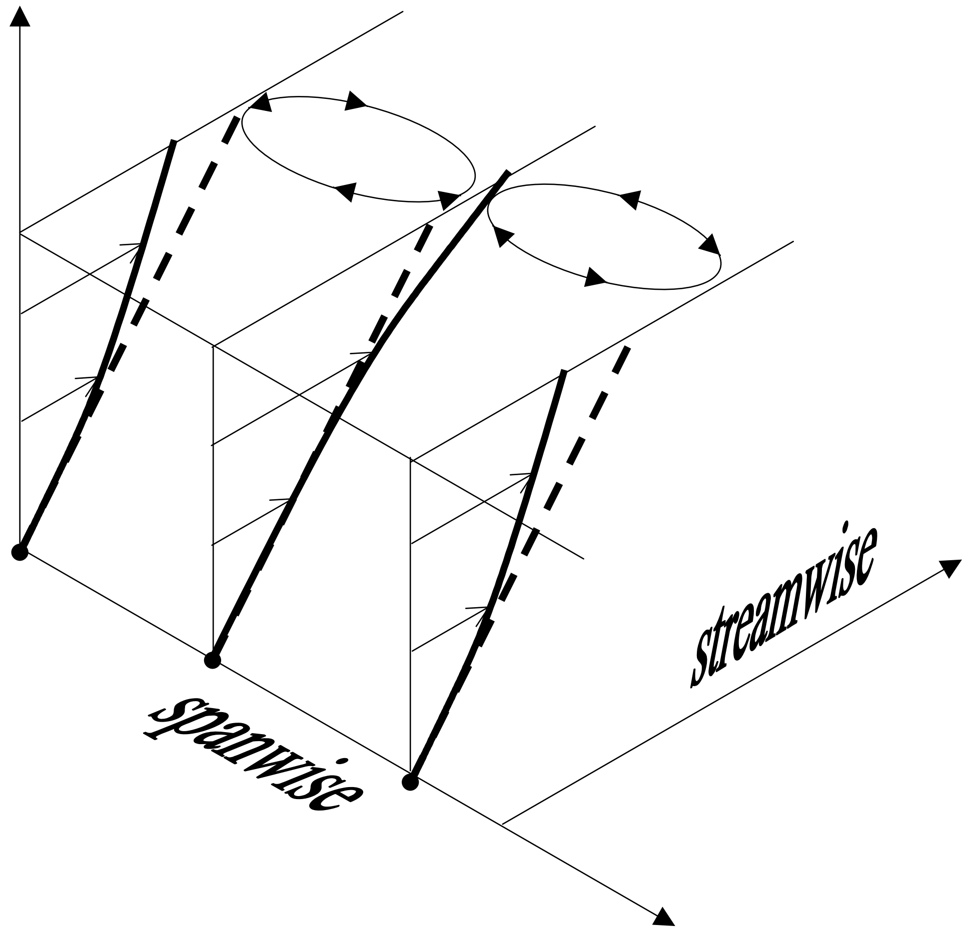
- Wall-normal vorticity fluctuations with spanwise modulation
- When towed by the flow: streaks

Voticity fluctuations with spanwise wave vector



Spanwise wave vector ($q_x = 0$), great simplification. Relatively simple analytical expressions are possible.

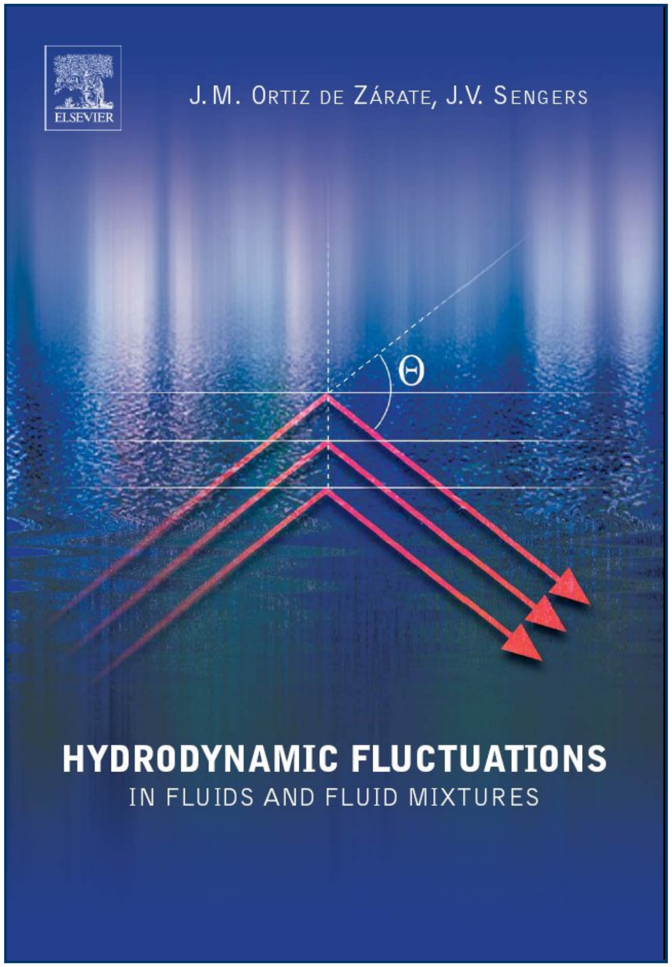
Thermal noise in real space



Generation of streaks

- Thermal noise adopts a streak form
- Streaks are maximally amplified

Thank you for your attention!!



To learn more. . .

- J. M. Ortiz de Zárate, J. V. Sengers
  Hydrodynamic Fluctuations in fluids and fluid mixtures
  Elsevier, 2006