Steady-State Two-Phase Flow in Porous Media: Open Questions

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Ground to be Covered:

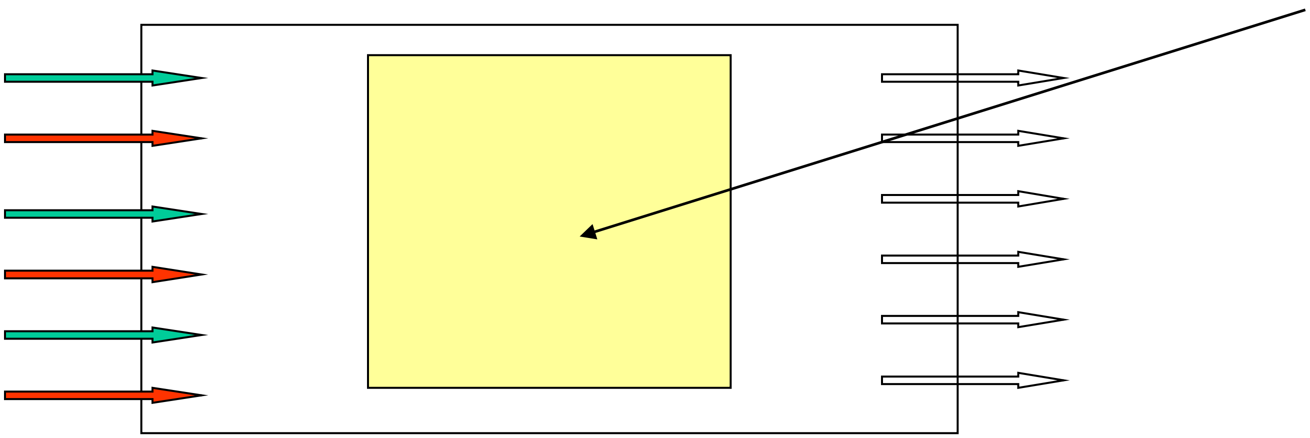
1. Steady-State Flow in the Laboratory
2. Steady-State Flow on the Computer
3. Nonlinear Rheology
4. Statistical Mechanics of Porous Media Flow

1. Steady-State Flow in the Laboratory

Steady-State Flow in the Laboratory

Both fluids move and fluid clusters break up and merge; still steady state.

A setup for studying steady-state flow in the laboratory:



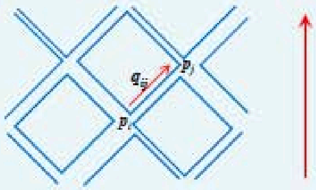
Region of *spatially homogeneous steady-state* flow.

Tallakstad et al., Phys. Rev. Lett. **102**, 074502 (2009);
Phys. Rev. E **80**, 036308 (2009).

2. Steady-State Flow on the Computer

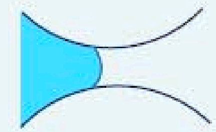
Numerical Model: Network of Connected Pores

- Disorder is incorporated by assigning the radius (r) of the tubes randomly, $r \in (0.1\ell, 0.4\ell)$.



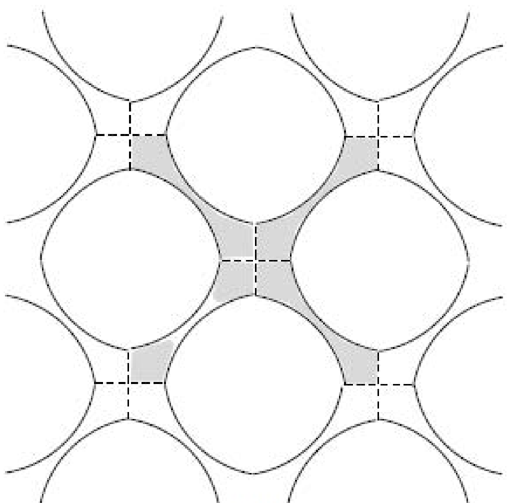
Flow in each tube obeys Washburn equation

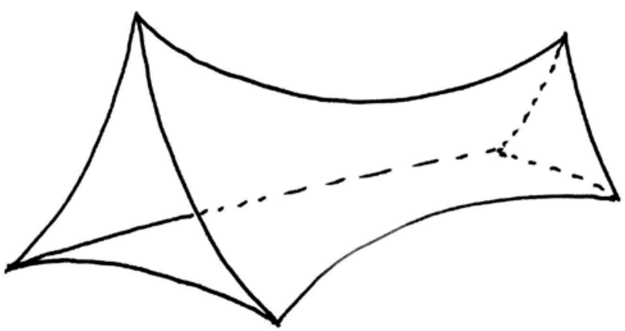
$$q_{ij} = -\frac{\pi r^4}{8\ell\mu_{eff}}\left(p_j - p_i - \sum p_c\right)$$



Tubes are hour-glass shaped with respect to the capillary pressure (p_c)

$$p_c = \frac{2\gamma}{r}\left(1 - \cos 2\pi x/\ell\right)$$


(a)



Knudsen et al. Transp. Por. Med. **47**, 99 (2002).

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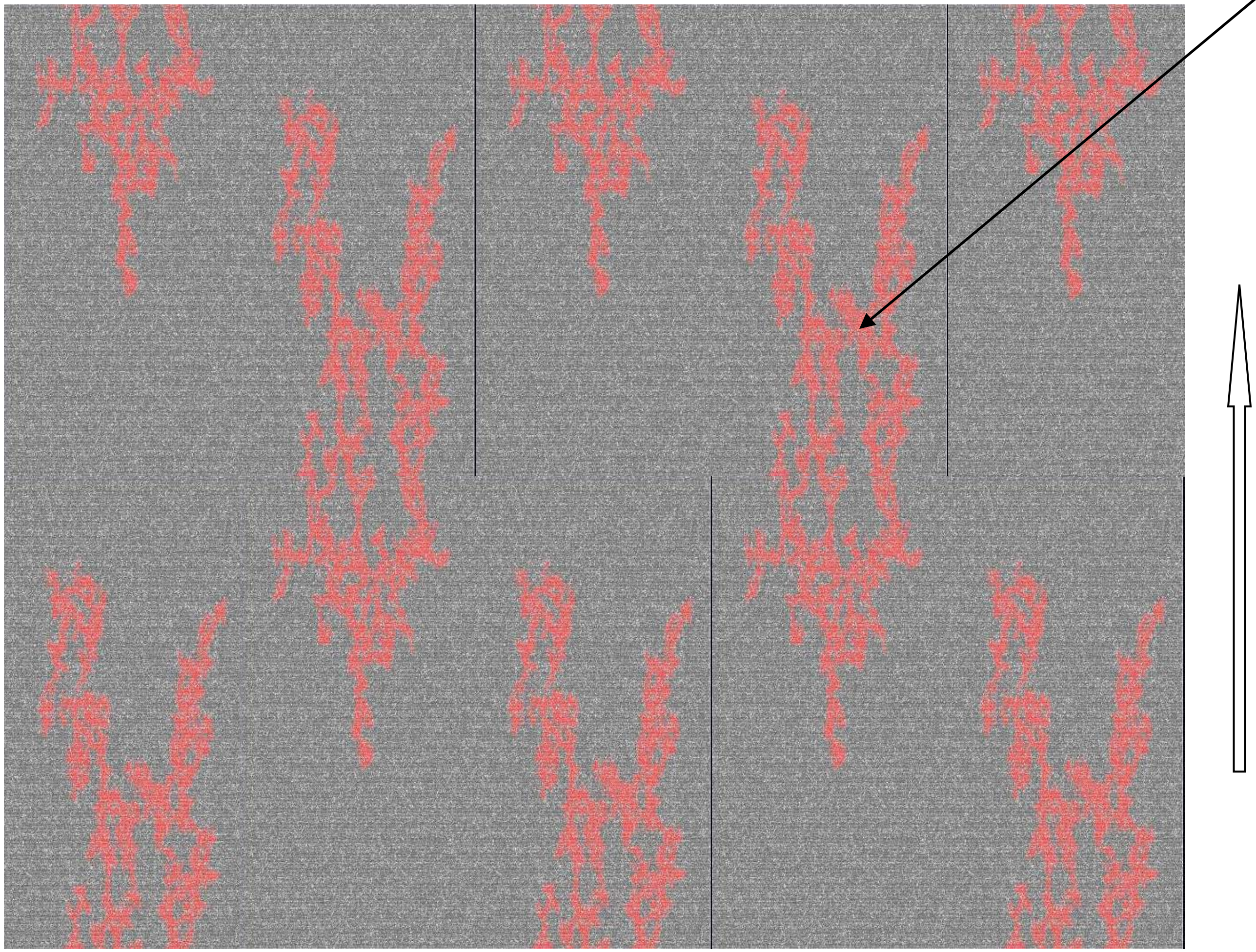
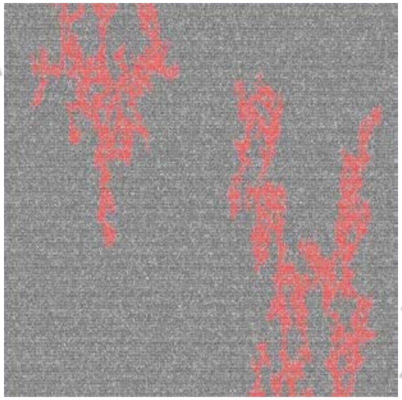
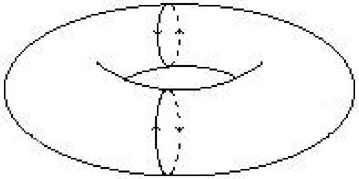
Steady-state Flow on a Torus



Largest non-wetting cluster.



Flow Direction



Hansen and Ramstad Comp. Geosci. **13**, 227 (2009)

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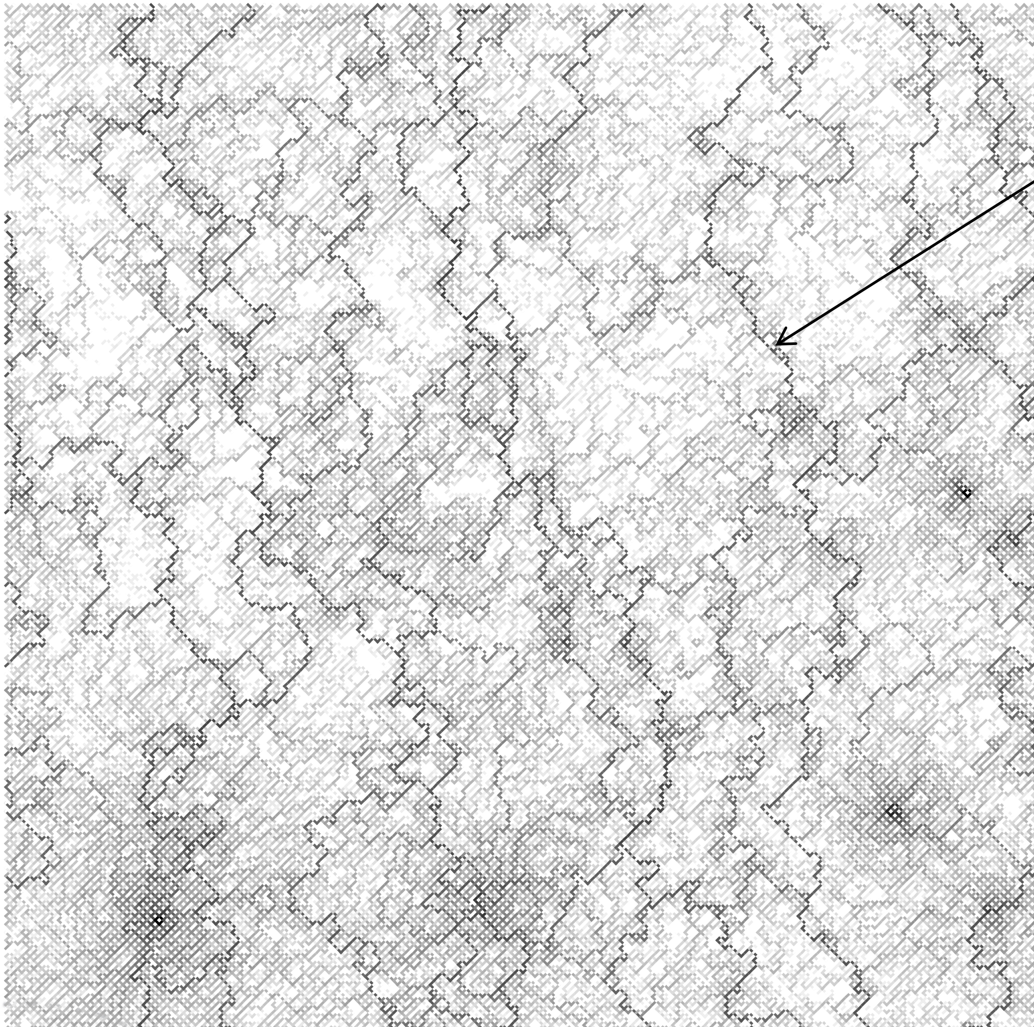
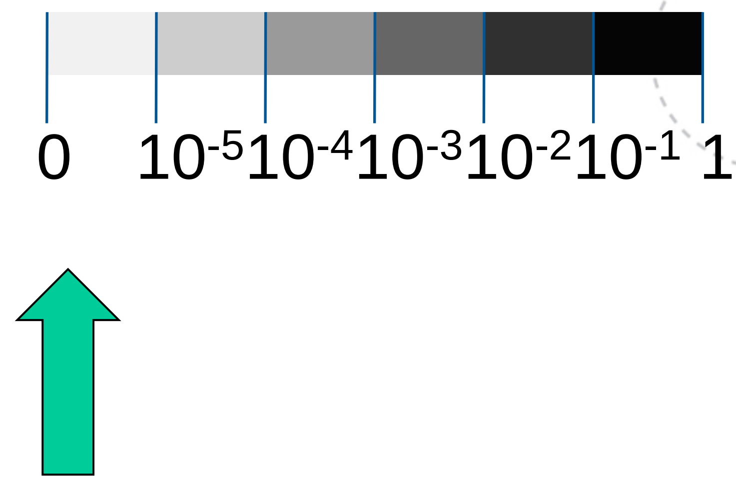
Flow Rate Distribution

Sinha, unpubl. (2011)



Local flow rates



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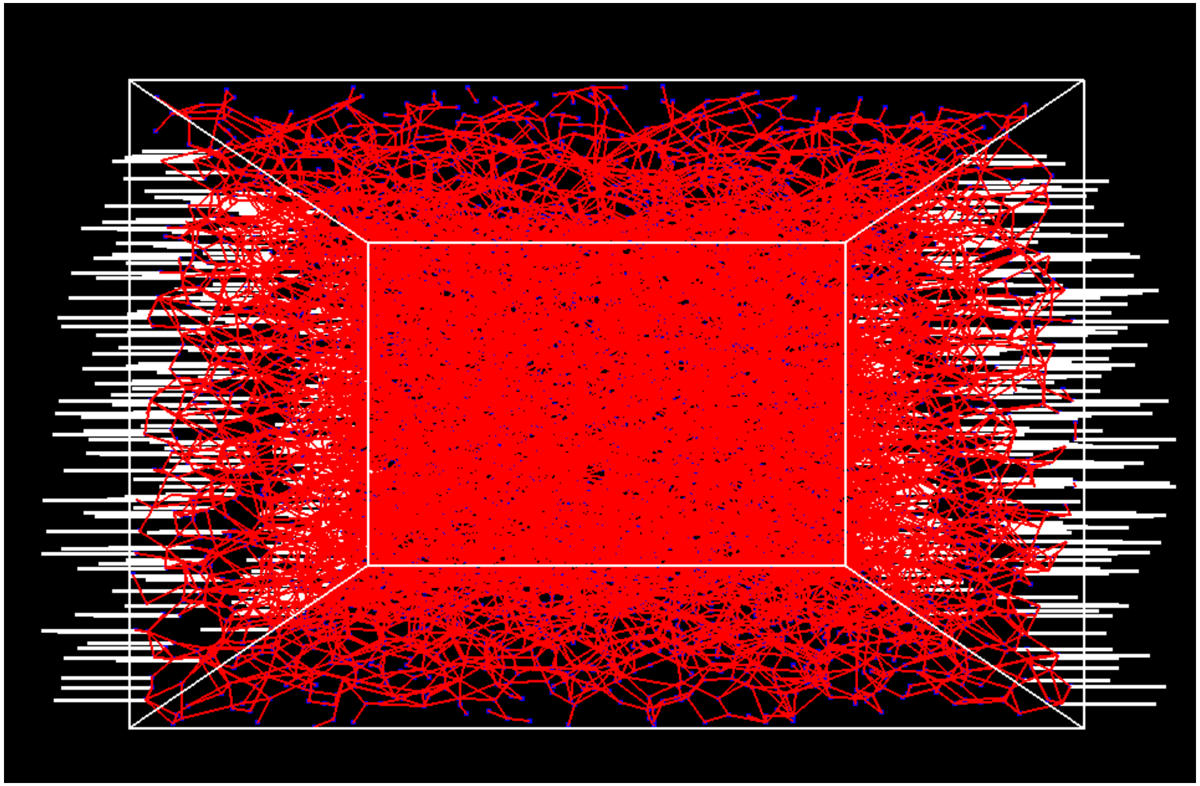
3 Dimensions: Reconstructed pore networks

Pore Network from Berea Sandstone

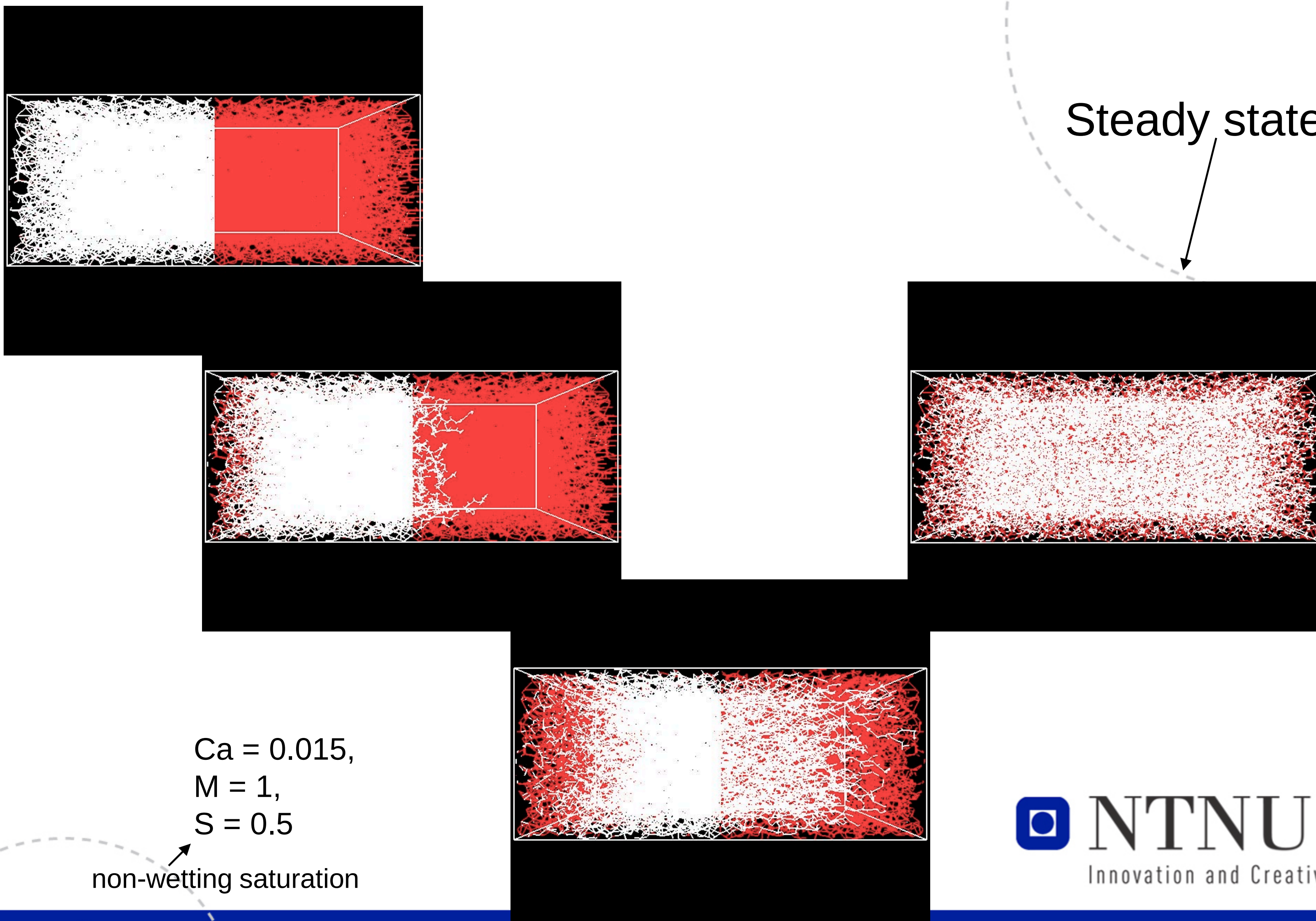


$(3mm)^3$

Each pore is described by a number of geometric parameters.

Reconstruction by e.g. merging thin slices

Evolution towards steady-state flow



Ca = 0.015,
M = 1,
S = 0.5

non-wetting saturation

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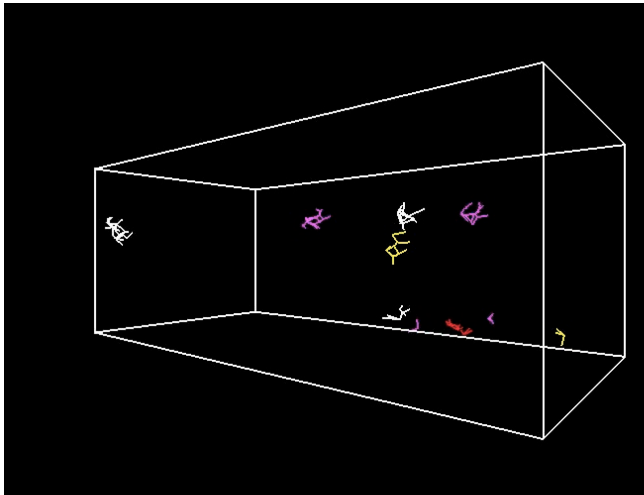
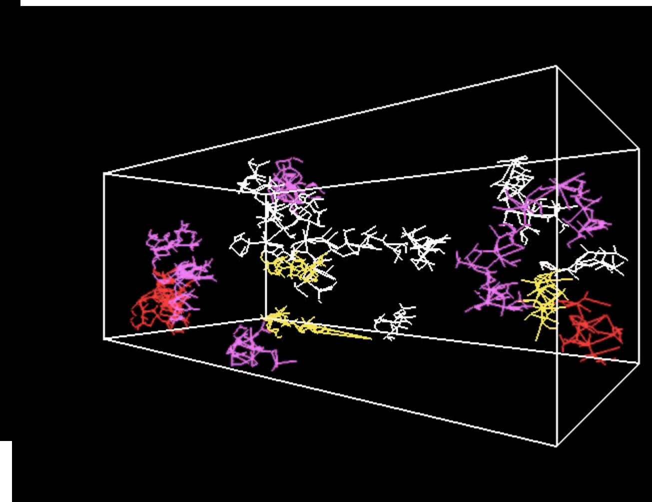
Largest non-wetting clusters at different saturation levels in steady state

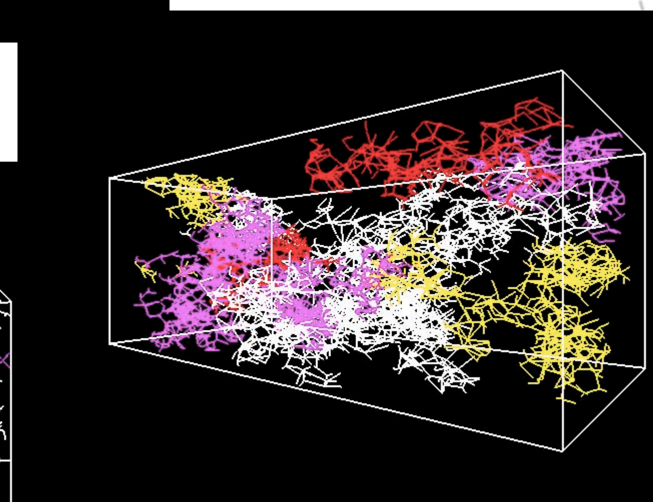
Ca=0.015, M=1



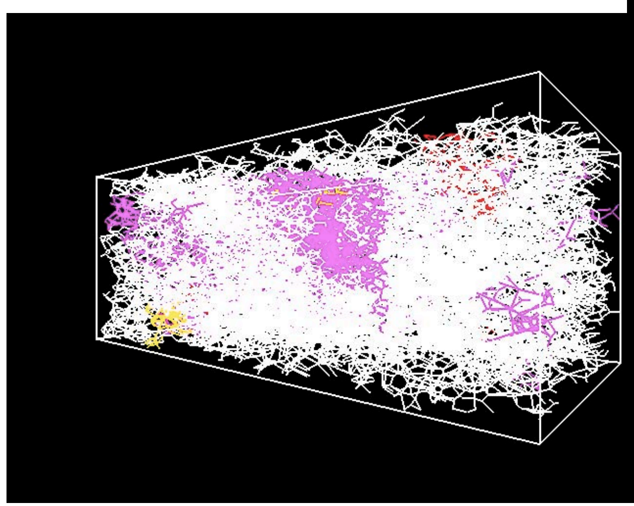
S=0.59



S=0.65



S=0.67



S=0.71

Critical saturation

Single vs. Two-Phase Flow (in 2D)

Knudsen and Hansen, Europhys. J. B **49**, 109 (2006)

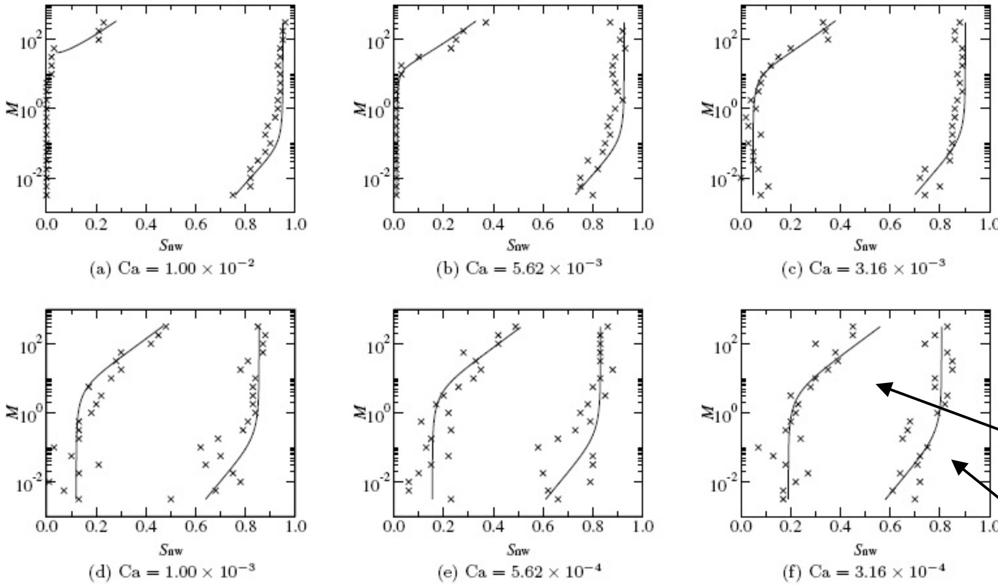


Fig. 3. The phase diagram for six selected values of Ca. The ×-marks are the transition points from the simulations. The uncertainty in each point is not marked, but it is substantial. The scattering of points in parameter space gives an indication of the uncertainty. Phase boundaries are indicated by solid lines, see Section 4. The phase diagrams are divided into three regions, counting from the left hand side: single-phase wetting flow, two-phase flow, and single-phase nonwetting flow.

Three-dimensional parameter space:

- Saturation S
- Viscosity Ratio M
- Capillary Number Ca

Both fluids move

Only one fluid moves

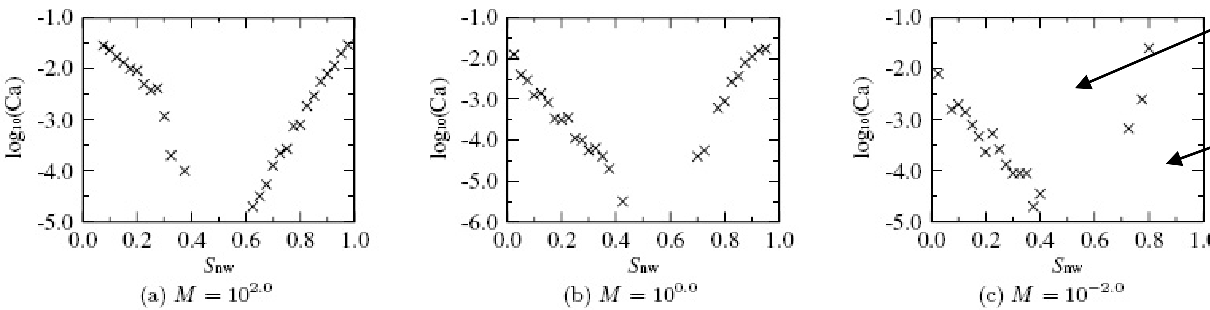


Fig. 4. The phase diagram for three selected values of M. The simulations were performed at constant saturation, but with varying Ca. The simulated points indicate the dynamical phase boundaries. In the lower left part of the diagrams, there is single-phase wetting flow; in the middle upper part, two-phase flow; and in the lower right part, single-phase nonwetting flow.

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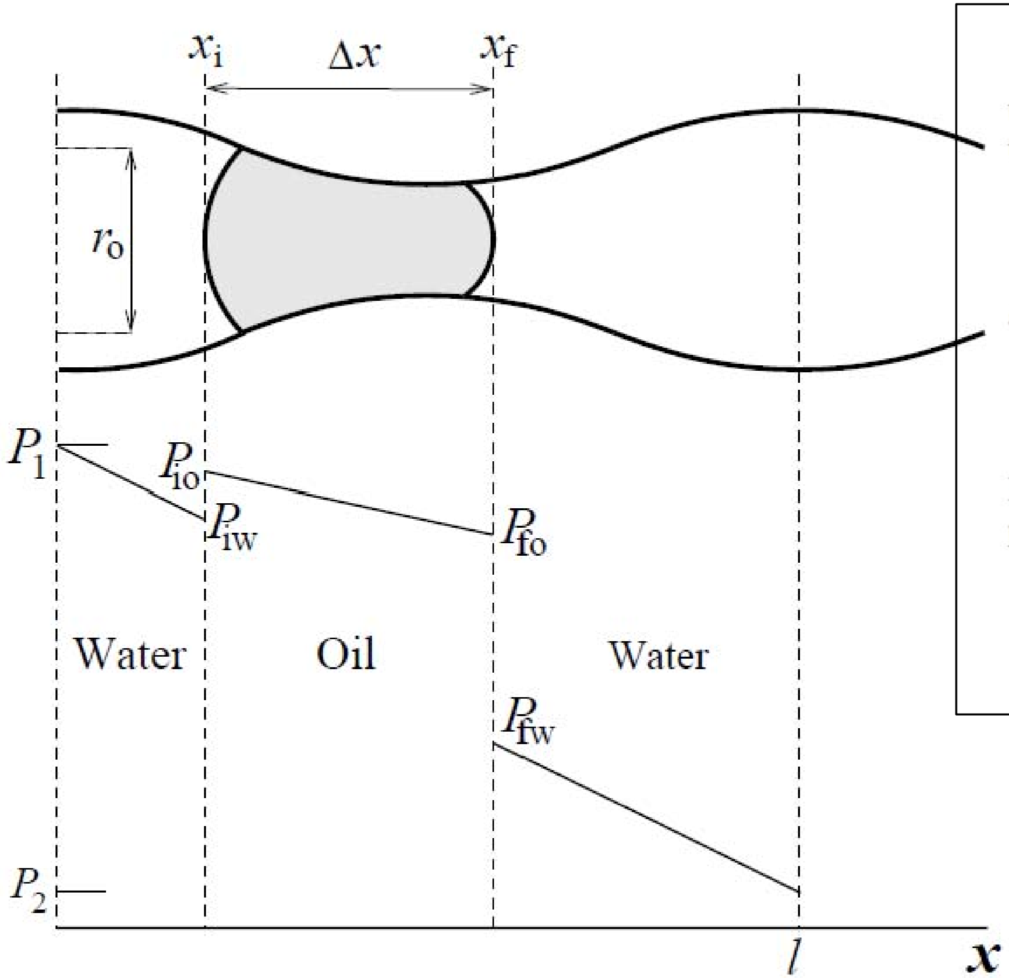
3. Nonlinear Rheology

Flow through a single tube



We now imagine a bubble in the tube limited by interfaces at x_i and x_f. The capillary pressure drop across the interface at x_i is

$$\frac{2\sigma}{r(x_i)} = \frac{2\sigma}{r_0}\left[1 - a\cos\left(\frac{2\pi x_i}{l}\right)\right], \tag{2}$$

and across the interface at x_f,

$$-\frac{2\sigma}{r(x_f)} = -\frac{2\sigma}{r_0}\left[1 - a\cos\left(\frac{2\pi x_f}{l}\right)\right], \tag{3}$$

Here, σ is the surface tension. We sum the two capillary pressure drops to get

$$p_c(x_b) = \frac{4\sigma a}{r_0}\ \sin\left(\frac{\pi \Delta x_b}{l}\right)\ \sin\left(\frac{2\pi x_b}{l}\right), \tag{4}$$

Sinha et al. to be submitted this week (2012)

$$q = -\frac{\pi r_0^4}{8\mu_{av} l} \left(\Delta p - p_c(x_b)\right)$$

Motion of bubble

$$\dot{x}_b = -\frac{r_0^2}{8l\mu_{av}} \left[\Delta p - \gamma \sin\left(\frac{2\pi x_b}{l}\right)\right],$$

This is the driven overdamped pendulum

$$\frac{d\theta}{d\tau} = \frac{|\Delta p|}{\gamma} + \sin\theta\ ,$$

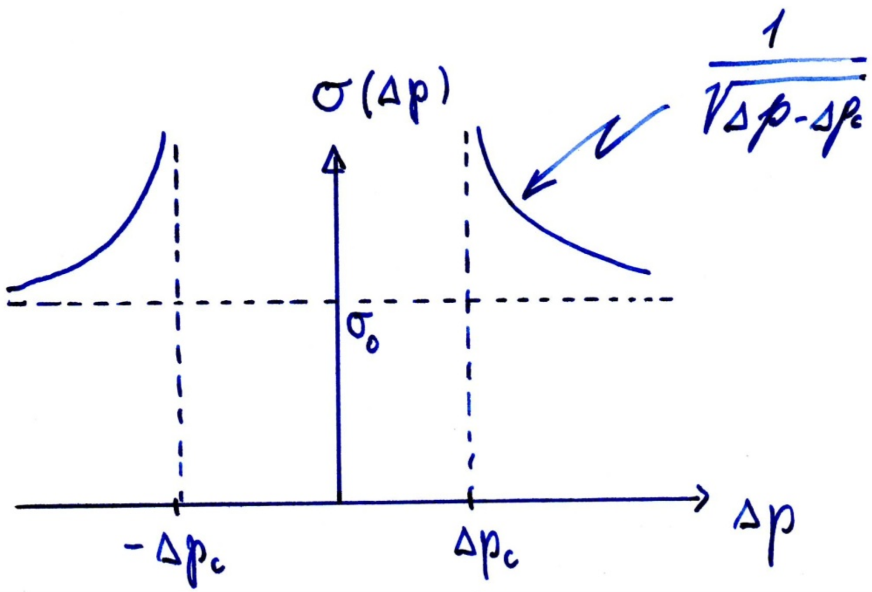
Effective response of single tube

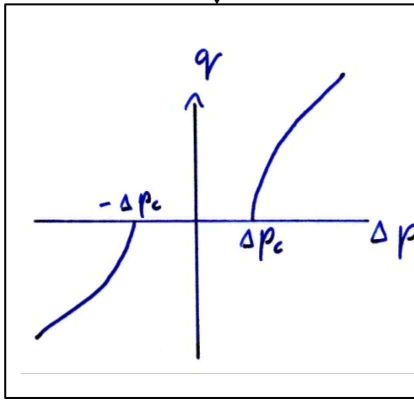
Saddle-node bifurcation

$$\langle q \rangle = -\frac{\pi r_0^4}{8\mu_{av} l}\,\mathrm{sgn}(\Delta p)\,\Theta(|\Delta p| - \gamma)\sqrt{\Delta p^2 - \gamma^2}\ ,$$

Effective conductivity

$$\sigma(\Delta p) = -\frac{dq}{d(\Delta p)} = \sigma_0 \begin{cases} \frac{|\Delta p|}{\sqrt{\Delta p^2 - \Delta p_c^2}} & \text{if } |\Delta p| > \Delta p_c\ , \\ 0 & \text{if } |\Delta p| \le \Delta p_c\ . \end{cases}$$





Experimental study of Steady-State flow in Hele-Shaw Cell

Tallakstad et al. Phys. Rev. Lett. **102**, 074502 (2009)

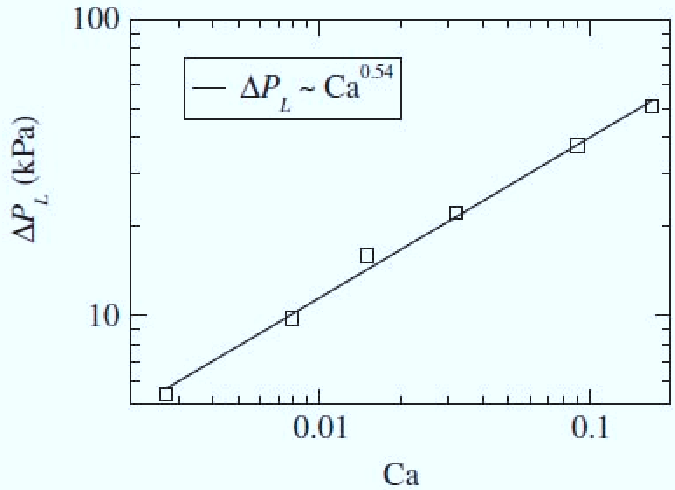


FIG. 2. Mean pressure difference ΔP_L during steady state as a function of Ca. The fluctuations in ΔP_L are of the order of 1 kPa, i.e., very small compared to the mean values. A power law dependence is found, with exponent $\beta = 0.54 \pm 0.08$.

Flow rate

$$\Delta P \propto Q^{0.54}$$

Pressure Drop

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Experiment in radial 3D geometry

Rassi et al. New J. Phys. **13**, 015007 (2011).

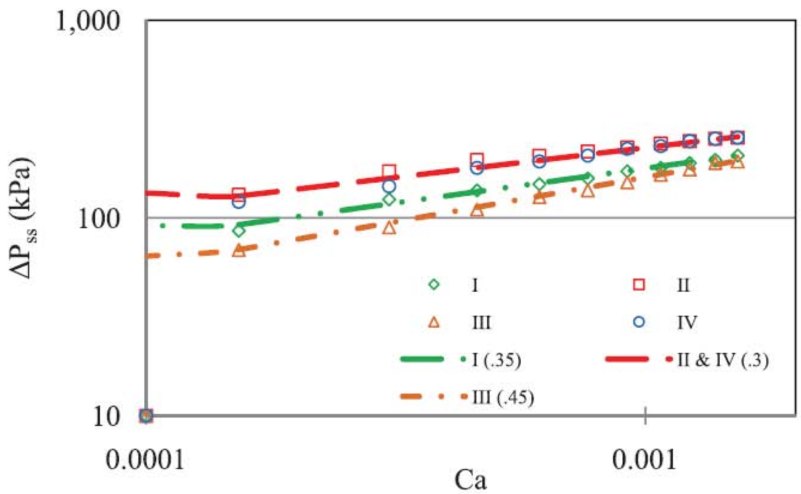

Figure 5. Average steady-state pressure drop versus capillary number for each repetition I–IV. The straight lines show the power-law fits for each repetition: I, $\beta = 0.35$; II, $\beta = 0.3$; III, $\beta = 0.45$; and IV, $\beta = 0.3$.

$$\Delta P \propto Q^{\beta}$$

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Behaves as a Bingham Plastic

"Newtonian" fluid with a yield threshold

Intuition: (Roux and Herrmann, Europhys. Lett. **4**, 1227 (1987).)

- Change pressure over network by $\delta(\Delta P)$.
- Number of additional links begin to flow: $\delta N \sim \delta(\Delta P)$.
- Conductance of network change by $\delta\Sigma \sim \delta N \sim \delta(\Delta P)$.
- Integrate to find $Q \sim (\Delta P - \Delta P_c)^2$.

Bingham plastic

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Effective medium theory

Generalized Darcy equation:

$$Q = -C\,\frac{A}{L}\,\frac{K(S_{nw})}{\mu_{\text{eff}}(S_{nw})}\,\text{sgn}\,(\Delta P)\,\begin{cases} (|\Delta P| - \Delta P_c(S_{nw}))^2 & \text{if } |\Delta P| > \Delta P_c\,, \\ 0 & \text{if } |\Delta P| \leq \Delta P_c\,, \end{cases}$$

Sinha and Hansen, Europhys. Lett., in press (2012).

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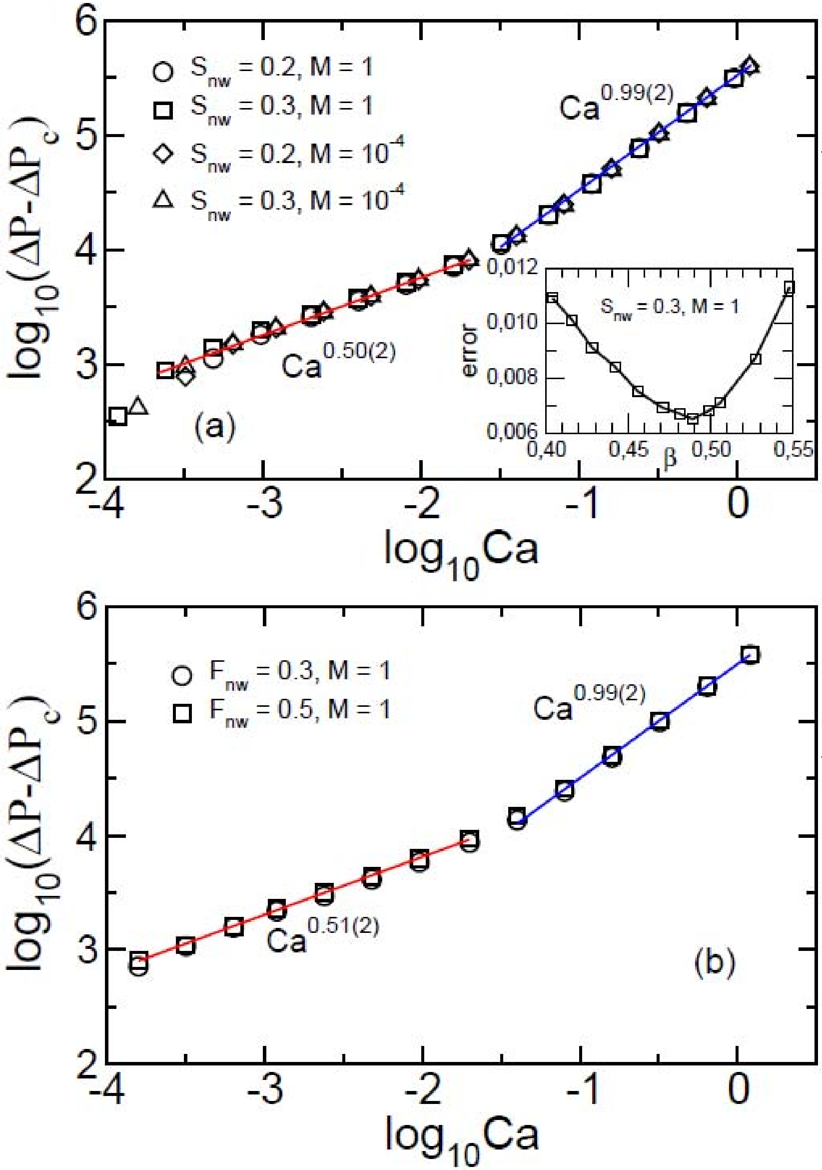
Numerical results



Torroidal boundary conditions

ΔP_c is independent of viscosity ratio M, but depends on saturation s.

Open boundary conditions

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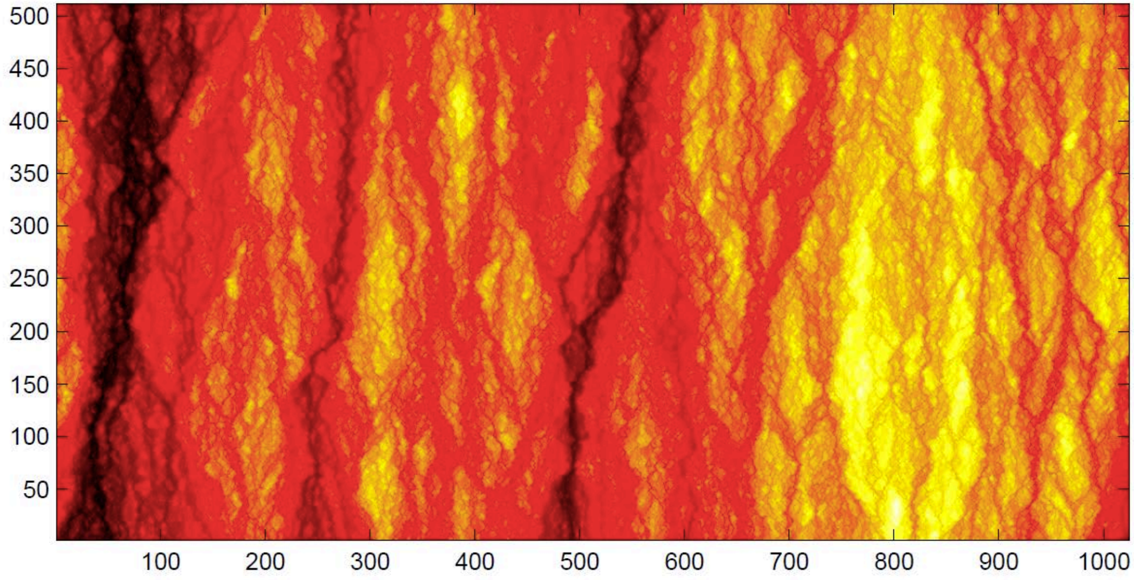
The threshold pressure ΔP_c

ΔP_c is independent of viscosity ratio M, but depends on saturation s.

Optimal path landscape:



Talon et al., in preparation (2012)

$$\Delta P_c = \min_{path} \Sigma_{i \in path} \Delta p_{c\,i}$$

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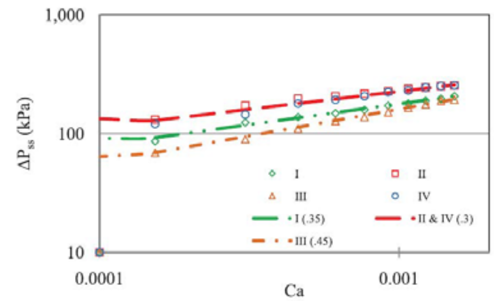
Reanalyzing the Rassi et al. data.



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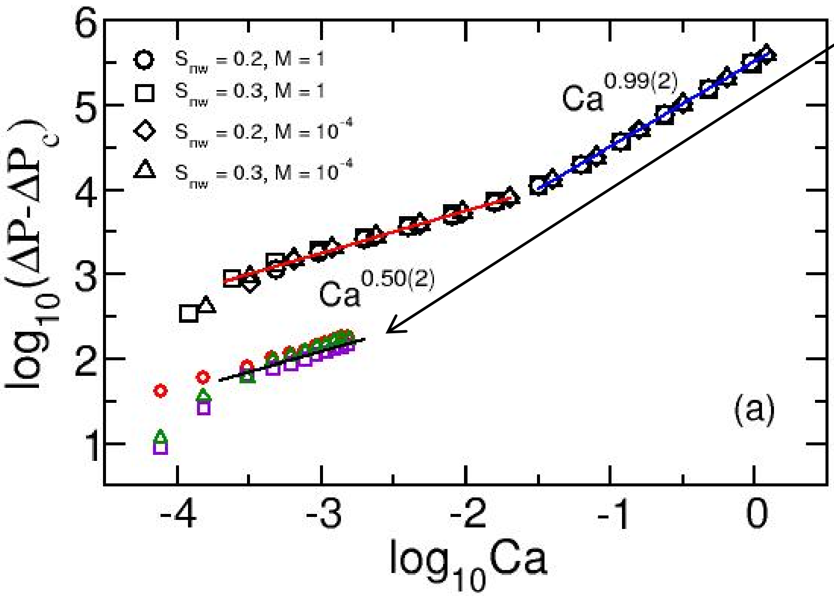

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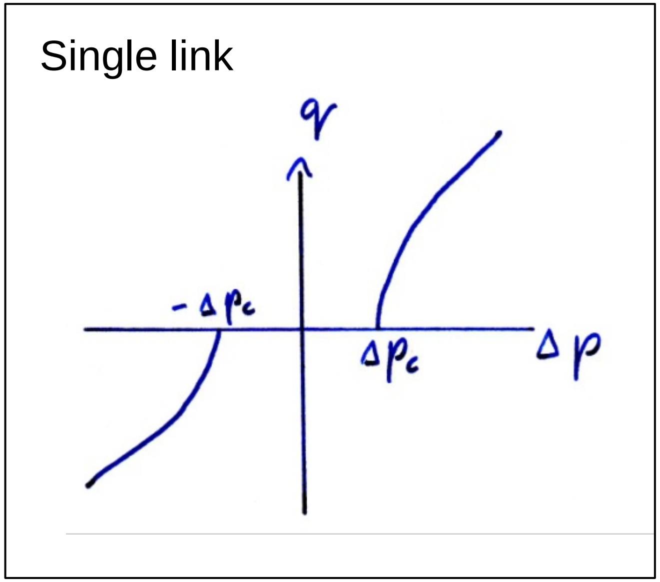
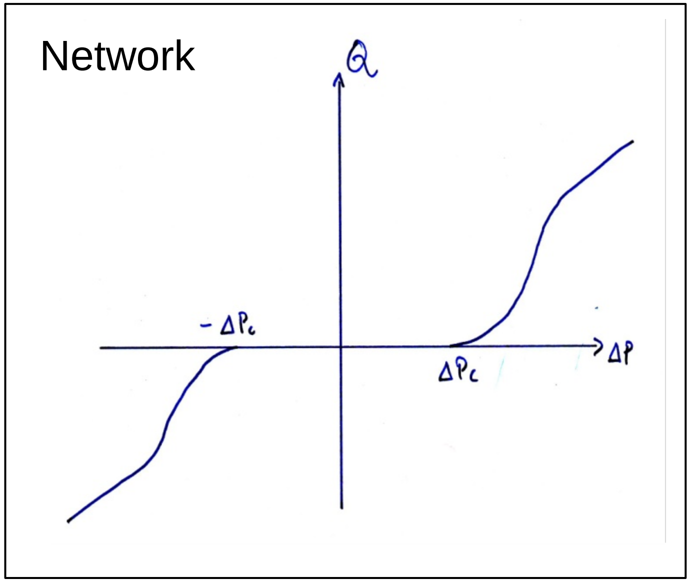
$$Q = -\, C\, \frac{A}{L}\, \frac{K(S_{nw})}{\mu_{\text{eff}}(S_{nw})} \text{sgn}\,(\Delta P) \begin{cases} (|\Delta P| - \Delta P_c(S_{nw}))^2 & \text{if } |\Delta P| > \Delta P_c\,, \\ 0 & \text{if } |\Delta P| \leq \Delta P_c\,, \end{cases}$$





$q \sim (\Delta p - \Delta p_c)^{1/2}$

$Q \sim (\Delta p - \Delta p_c)^2$

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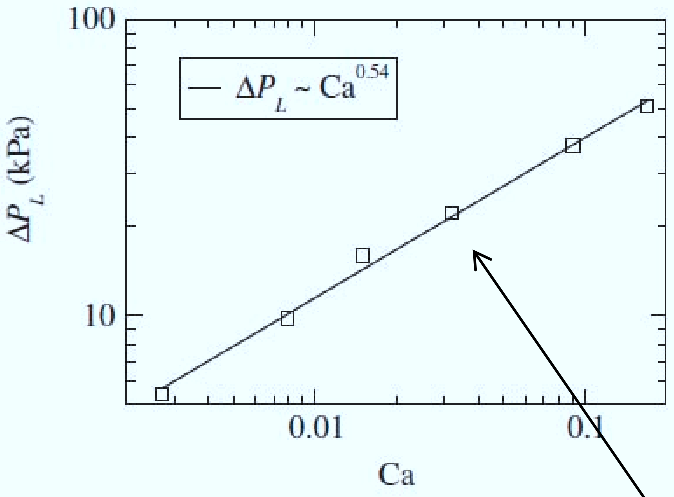

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$$\Delta P \propto Q^{0.54}$$

vs.

$$\Delta P - \Delta P_c \propto Q^{1/2}$$

System prepared so that one of the fluids percolates: $\Delta P_c \approx 0$.

(There *is* some curvature)

4. Statistical Mechanics of Porous Media Flow

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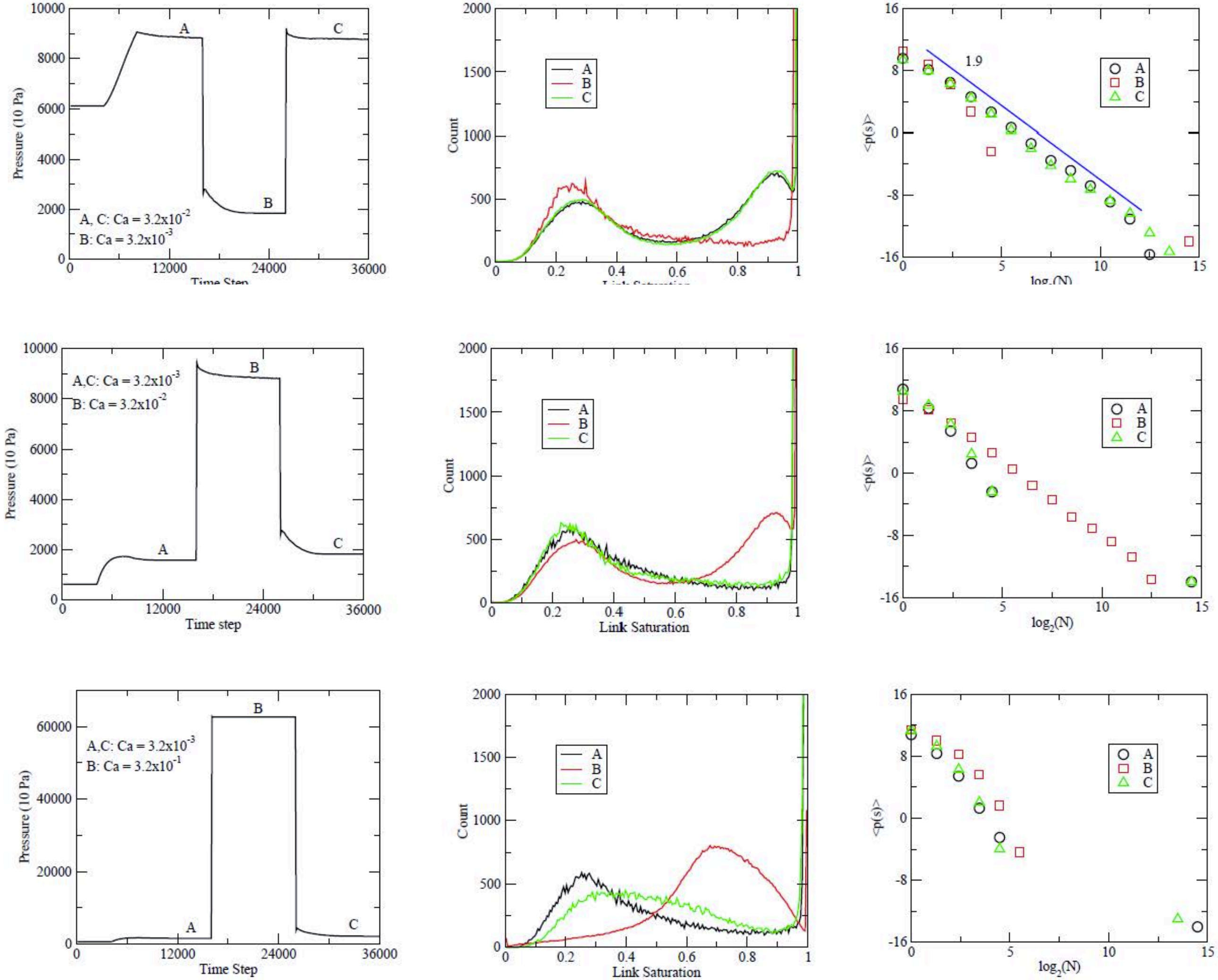
Returning to the concept of a state.

Sinha, 2012



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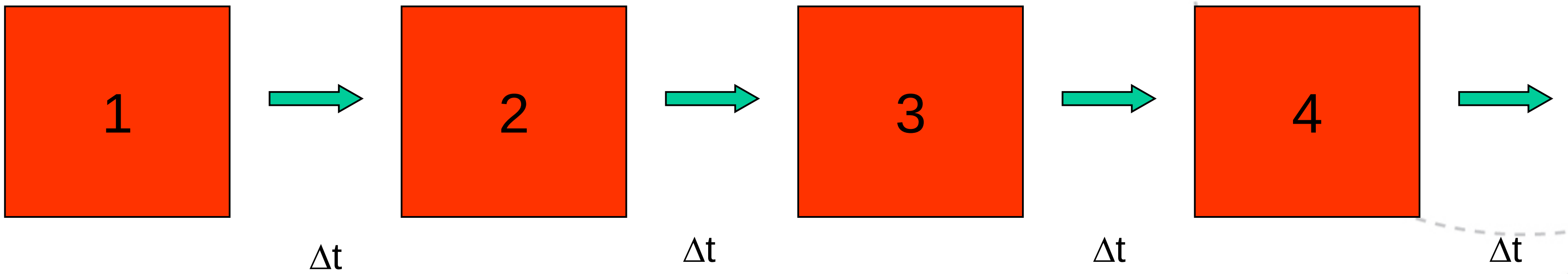
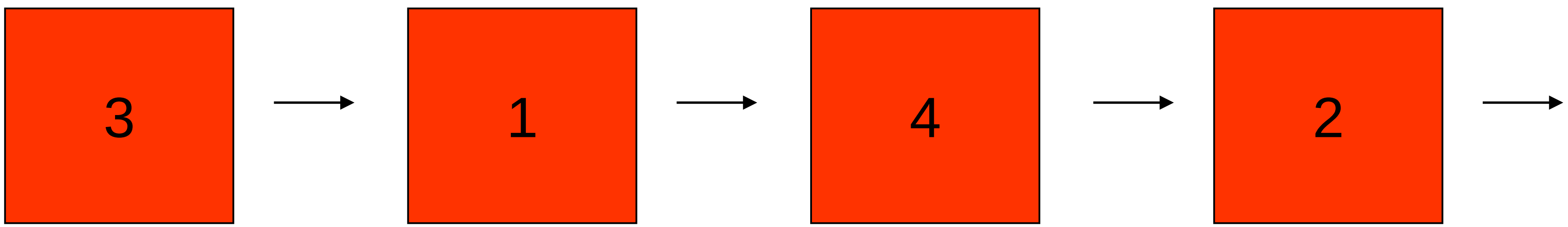
Sequence of configurations through time integration:



The order of the configurations has been randomized:



This randomization does not change the statistics.

If order plays no role: All steady-state properties will be completely described by the configurational probability distribution $\Pi\{cf\}$ where {cf} signifies the positions of all interfaces between the immiscible fluids in the porous medium.

A configuration is fully described by the position of all interfaces.

This leads to a statistical mechanics for porous media.

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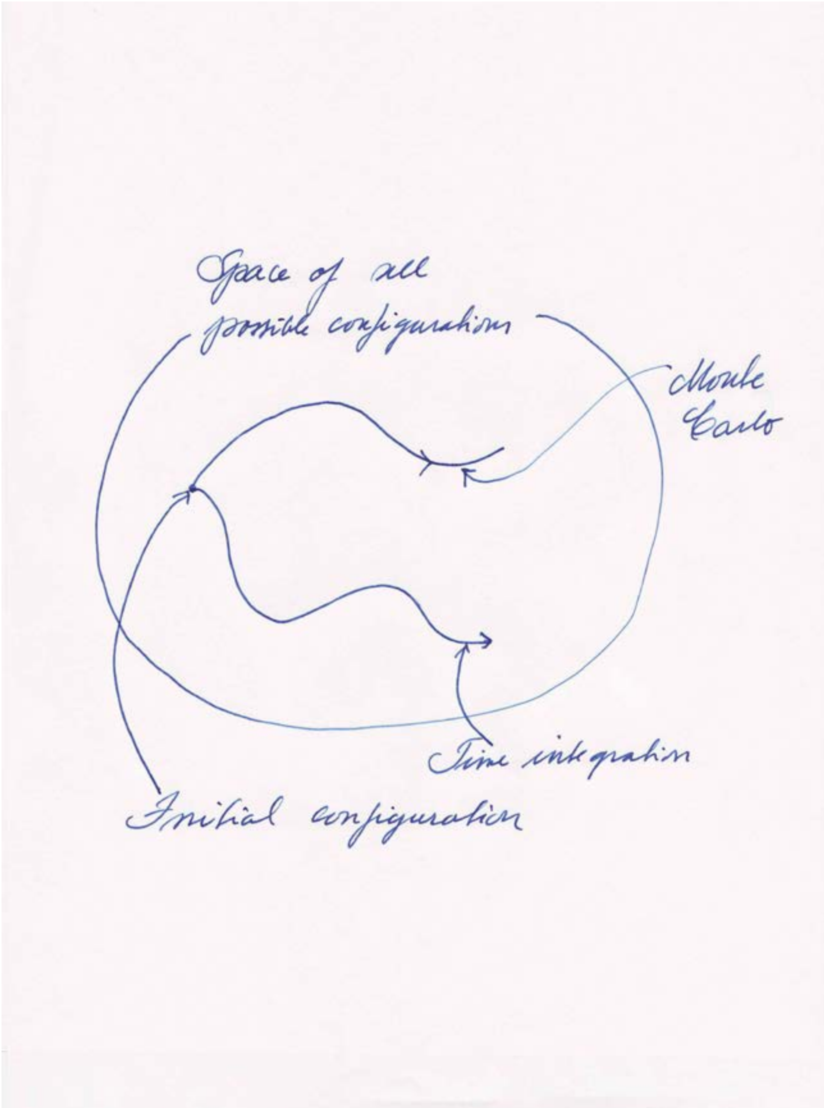
Metropolis Monte Carlo Sampling



Configurational probability

$\Pi\{cf\}$

Old configuration $\{cf_{old}\}$ ⇨ Test configuration $\{cf_{test}\}$

↑ Chosen by random change of old configuration.

Draw a random number $r \in [0,1]$.

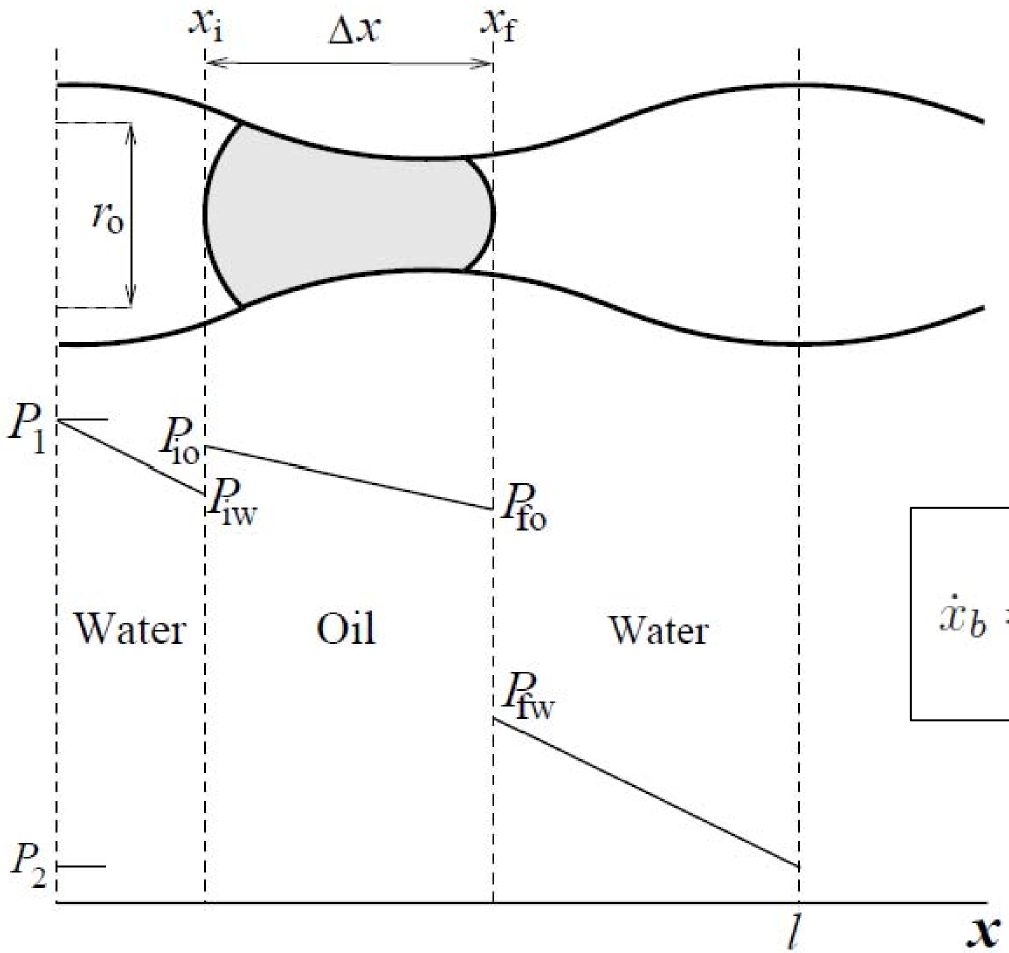
If $\Pi\{cf_{old}\}/\Pi\{cf_{test}\} > r$: Reject test configuration.

If $\Pi\{cf_{old}\}/\Pi\{cf_{test}\} \leq r$: Accept test configuration.

Hansen and Ramstad Comp. Geosci. **13**, 227 (2009)

Can we derive Π{cf}?

Flow through a single tube



$$q = -\frac{\pi r_0^4}{8\mu_{av} l} \left(\Delta p - p_c(x_b)\right)$$

$$\dot{x}_b = -\frac{r_0^2}{8 l \mu_{av}} \left[\Delta p - \gamma \sin\left(\frac{2\pi x_b}{l}\right)\right] ,$$

Sinha et al. to be submitted this week (2012)

$f(x_b)$ is some function of x_b.

$$\langle f \rangle = \frac{1}{T}\int_0^T f\left(x_b(t)\right) dt = \int_0^l f\left(x_b\right) \frac{1}{T}\frac{dt}{dx_b}\, dx_b$$

$$= \int_0^l f\left(x_b\right) \frac{\pi r_0^2}{T}\frac{1}{q(x_b)}\, dx_b = \int_0^l f\left(x_b\right) \Pi\left(x_b\right) dx_b$$

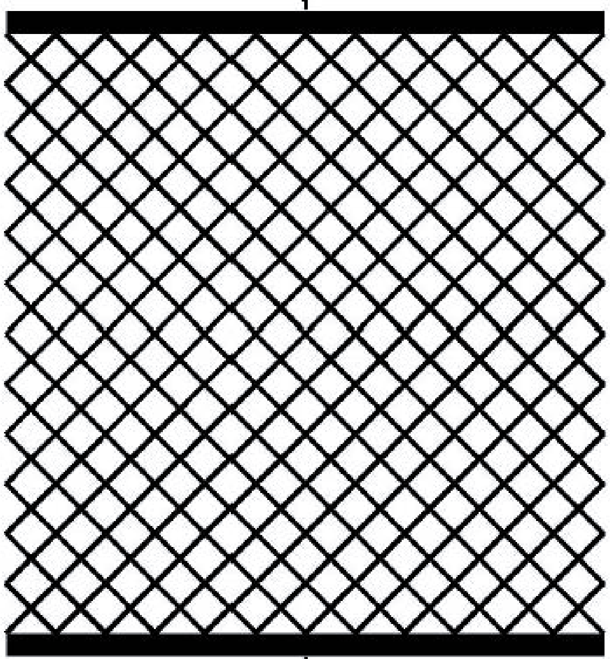
$$\Pi\left(x_b\right) = \frac{\pi r_0^2}{T}\frac{1}{q(x_b)}$$

Configurational probability $\Pi\{cf\}$ where cf = x_b.

$$q(x_b) = \frac{\pi r_0^4}{8\mu_{av} l}\left(\Delta p - p_c(x_b)\right)$$

Π{cf} for a porous medium.

Q



Porous medium

Connected in series

$$\Pi\left\{\mathrm{cf}\right\} = \Pi\left(x\right) = \frac{\pi r^2}{T}\frac{dt}{dx} = \frac{\pi r^2}{T}\frac{1}{Q(x)}$$

Tube

x

Follow a point x in time.

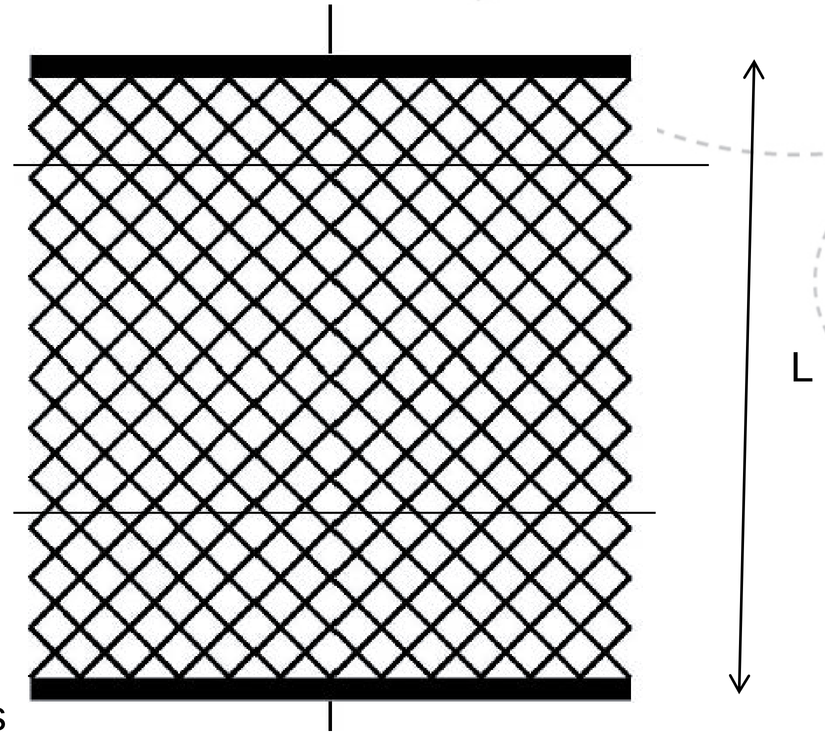
System is deterministic: cf = cf(x)

Q

Microcanonical ensemble:

$$\Pi\{\mathrm{cf}\} = \frac{\pi r^2}{T} \frac{1}{Q\{\mathrm{cf}\}}$$

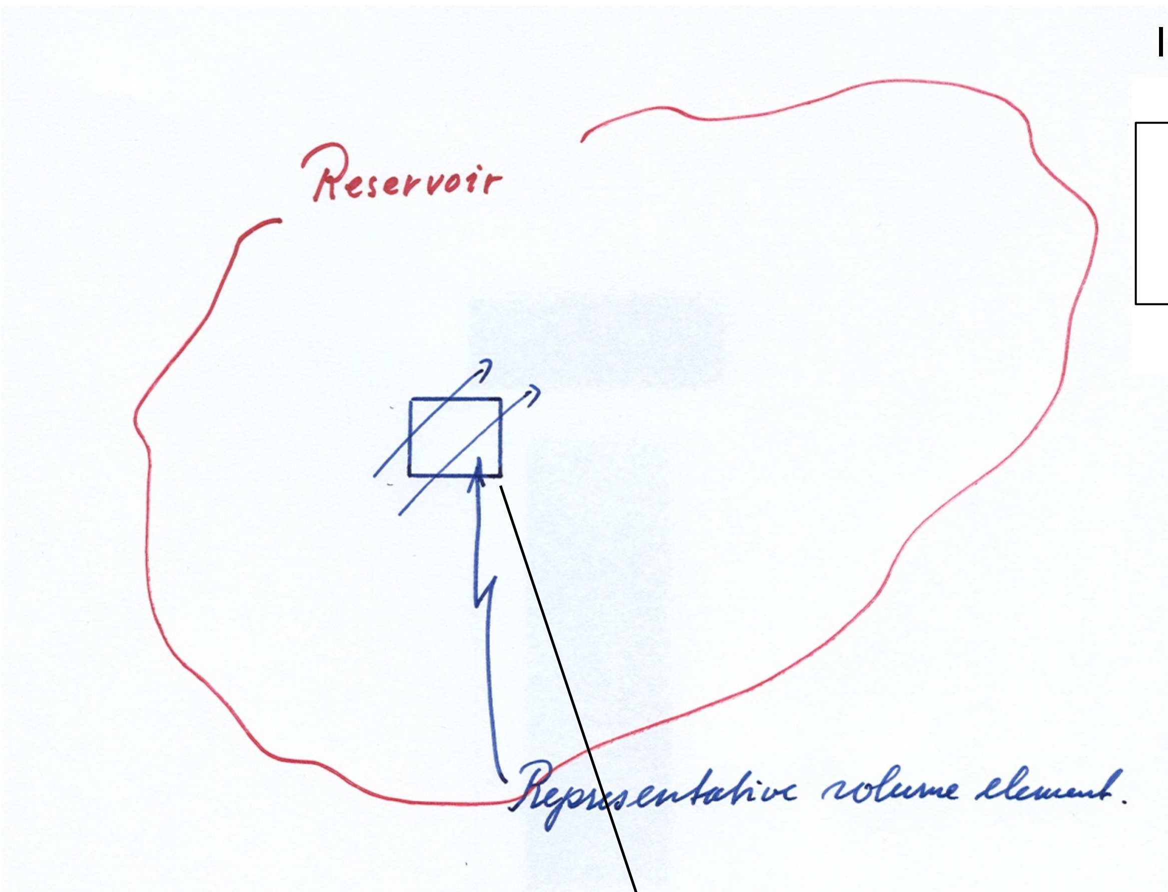
Another expression for Q:



$$Q = \frac{1}{L} \sum_{<i,j>} q_{i,j}$$

Q is sum of currents
in the bonds that are cut by the line.

Sum over all bonds in network.



Isolated system:

$$\Pi\{\mathrm{cf}\} = \frac{\pi r^2}{T}\frac{1}{Q\{\mathrm{cf}\}}$$

$$Q = \frac{1}{L}\sum_{<i,j>} q_{i,j}$$

Boltzmann-like distribution: $\Pi\{\mathrm{cf}\} = \exp[-Q\{\mathrm{cf}\}/T]$

Configurational temperature

Summary:

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