A comparative analysis of efficiency a maximum power output for some heat engines

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Abstract

In this work, we present a comparative analysis of the efficiencies at maximum power output for different devices; first, we analyze the expressions of the efficiency at maximum power (EMP) and maximum ecological function conditions (EME), for two models of heat engines: a model of Curzon-Ahlborn (macroscopic) and a Novikov model, in these two cases different heat transfer laws were considered: A heat transfer due to free conduction described by Newton's law of cooling, a phenomenologic heat transfer law comes from linear irreversible thermodynamics and a nonlinear heat transfer law (Dulong-Petit). We compare these efficiencies with the results presented recently by Esposito et al. published in Phys. Rev. Lett. 105, 150603 (2010). For our comparison, we expand the expressions of efficiencies up to the third-order term of de Carnot efficiency. We found some interesting similarities for the expressions of efficiencies between the macroscopic and mesoscopic models of heat engines.

Optimal efficiencies both maximum power output and maximum ecological function conditions

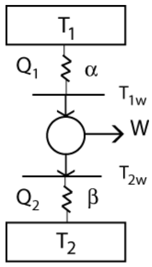


Fig. 1 Curzon-Ahlborn heat engine

$$Q_1 = \alpha(T_1 - T_{1w})^k, \qquad \gamma = (\alpha/\beta)^{1/k}$$

$$Q_2 = \beta(T_{2w} - T_2)^k. \qquad \tau = T_2/T_1 \qquad k \geq 1$$

$$W(\eta,k) = \alpha T_1^k \eta \left[\frac{1-\eta-\tau}{\gamma(1-\eta)^{1/k} + (1-\eta)}\right]^k, \quad \eta = 1 - T_{2w}/T_{1w}$$

$$E(\eta,k) = \alpha T_1^k \left[2\eta - (1-\tau)\right]\left[\frac{1-\eta-\tau}{\gamma(1-\eta)^{1/k} + (1-\eta)}\right]^k,$$

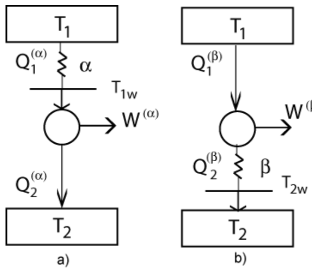


Figure 2. Novikov heat engines

$$\eta_{MP}^{(\alpha)}(\tau,k) = \left[1 - \frac{\tau}{2}(1-k)\right] - \sqrt{\left[1 - \frac{\tau}{2}(1-k)\right]^2 + (\tau - 1)}$$

$$\eta_{MP}^{(\beta)}(\tau,k) = \frac{1}{k}\left[\frac{1+k}{2} - \sqrt{k\tau + \left(\frac{1-k}{2}\right)^2}\right]$$

Model	$k=1$	$k=5/4$	$Q \propto \Delta T^{-1}$ [Chen]
Fig 1 a)	$\eta_{MP} = 1-\sqrt{\tau}$	No analytical	$1 - \frac{1+3\tau}{3+\tau}$
Fig 1 b)	$\eta_{MP}^{\alpha} = 1-\sqrt{\tau}$	$\frac{8+\tau-\sqrt{\tau(\tau+80)}}{8}$	$\frac{1}{2}(1-\tau)$
Fig 1 c)	$\eta_{MP}^{\beta} = 1-\sqrt{\tau}$	$\frac{9-\sqrt{1+80\tau}}{10}$	$\frac{1-\tau}{1+\tau}$

$$k=1 \qquad \eta_{CA} = \frac{\eta_C}{2} + \frac{\eta_C^2}{8} + \frac{3\eta_C^3}{96} + O(\eta_C^4)$$

$$\eta_{MP}^{(\alpha)} = \frac{4\eta_C}{9} + \frac{100\eta_C^2}{729} + \frac{4100\eta_C^3}{59049} + O(\eta_C^4)$$

$$k = 5/4$$

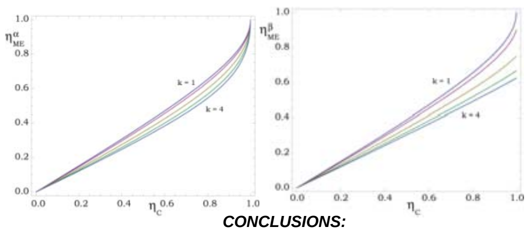
$$\eta_{MP}^{(\beta)} = \frac{4\eta_C}{9} + \frac{80\eta_C^2}{729} + \frac{3200\eta_C^3}{59049} + O(\eta_C^4)$$

$$\gamma \to 1 \qquad \eta_{MP(1)}^{(-1)} = \frac{\eta_C}{2} + \frac{\eta_C^2}{8} + \frac{3\eta_C^3}{96} + O(\eta_C^4)$$

$$\gamma \to \infty \qquad \eta_{MP(\infty)}^{(-1)} = \frac{\eta_C}{2} + O(\eta_C^4) \qquad Q \propto \Delta T^{-1}$$

$$\gamma \to 0 \qquad \eta_{MP(0)}^{(-1)} = \frac{\eta_C}{2} + \frac{\eta_C^2}{4} + \frac{\eta_C^3}{8} + O(\eta_C^4)$$

For the efficiency under maximum ecological conditions:



Efficiency at maximum power for low-dissipation Carnot-like engines -Phys. Rev. Lett. 105, 150603 (2010)-

The amount of heat exchange per cycle and the power output could be written as:

$$Q_c = T_c\left(-\Delta S - \frac{\Sigma_c}{\tau_c} + \ldots\right), Q_h = T_h\left(\Delta S - \frac{\Sigma_h}{\tau_h} + \ldots\right) \text{ and } P = \frac{(T_h - T_c)\Delta S - T_h \Sigma_h/\tau_h - T_c \Sigma_c/\tau_c}{\tau_c + \tau_h}$$

The maximum power is found by setting the derivatives of P with respect to τ_h and τ_c equal zero,

$$\eta^* = \frac{\eta_C\left(1 + \sqrt{\frac{T_c \Sigma_c}{T_h \Sigma_h}}\right)}{\left(1 + \sqrt{\frac{T_c \Sigma_c}{T_h \Sigma_h}}\right)^2 + \frac{T_c}{T_h}\left(1 - \frac{\Sigma_c}{\Sigma_h}\right)}, \text{ when } \Sigma_h = \Sigma_C \text{ then } \eta_{MP}^{EK} = \frac{\eta_C}{2} + \frac{\eta_C^2}{8} + \frac{6\eta_C^3}{96} + O(\eta_C^4)$$

This result was also obtained by Schmiedl and Seifert for the stochastic heat engine (EPL 81 (2008)) the same expression for Curzon-Ahlborn efficiency, obtained for heat engine of figure 1 when a Newtonian heat transfer low is considered. In the limits $\Sigma_c/\Sigma_h \to 0$ and $\Sigma_c/\Sigma_h \to \infty$, the efficiency at maximum power converges to the upper bound $\eta_+ = \eta_C/(2-\eta_C)$ and to the lower bound $\eta_- = \eta_C/2$, and their expression for efficiency in expansion series are:

$$\eta_{MP}^{EK} = \frac{\eta_C}{2} + \frac{\eta_C^2}{4} + \frac{\eta_C^3}{8} + O(\eta_C^4) \qquad \text{and} \qquad \eta_{MP}^{EK} = \frac{\eta_C}{2} + O(\eta_C^4)$$

CONCLUSIONS:

In this work we presented a briefly review of some recent results published about EMP of heat engines, we mentioned the main considerations to obtained the efficiency at maximum power for each heat engine model, we chose these models because the EMP in some case coincide in spite of their nature, macroscopic or mesoscopic, and where a specific transfer laws is considered and where it is not. It is possible to talk about universality of EMP?, or may be there is a universal conditions for heat engine for which the CA efficiency always appeared as in Carnot case, where reversible conditions are necessary have it, independently of working fluid?

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