# SIMPLE METHOD FOR DETERMINING STRESS AND STRAIN CONSTANTS FOR NON-STANDARD MEASURING SYSTEMS ON A ROTATIONAL RHEOMETER

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#### ABSTRACT:

There is often a necessity to measure, or at least estimate, true viscosity values using non-standard measuring systems on a rotational rheometer. This may be to replicate a mixing or manufacturing process on a lab scale, to keep a sample dispersed and uniform during a measurement or to measure some rheological property that would be difficult or impossible with a standard configuration. Such measurements can be made easily enough, but without a process for converting torque to shear stress and angular velocity to shear rate only these raw data variables can be reported. In this paper a simple and novel empirical method for determining strain/strain rate  $C_1$  and stress  $C_2$  constants for non-standard measuring systems on a rotational rheometer is presented. This method uses relative torque measurements made with a Newtonian and non-Newtonian material and their corresponding power law fitting parameters to determine  $C_1$  and  $C_2$  using a non-linear regression analysis. Equilibrium flow curves generated for two non-Newtonian fluids using two non-standard mixing geometries show very good agreement with data generated using a standard cone and plate configuration, therefore, validating the approach.

#### KEY WORDS:

Measuring system constant, true viscosity, torque, mixer, angular velocity, shear rate factor

## **1** INTRODUCTION

To measure shear viscosity accurately using a rotational rheometer it is important that the flow field is homogenous and laminar and the shear rate is well defined. For this to be the case the shear gap needs to be small (based on the sample being measured) and linearly dependent on the velocity at the shearing surface. Such conditions are met in the case of 'cone and plate' measuring systems, so long as the angle between the plate and cone is small, because any increase in linear velocity with cone radius correlates with an equivalent increase in shear gap. The working equations for cone and plate are shown below (Equations 1 and 2) with  $\omega$  the angular velocity,  $\theta$  the cone angle,  $\tau$  the torque, and R the radius of the cone.

$$\dot{\gamma} = \frac{\omega}{\theta} \tag{1}$$

$$\sigma = \frac{3\tau}{2\pi R^3}$$

For a 'parallel plate' system the linear velocity increases with radius, but since the gap remains constant the shear rate then varies across the radius of the plate [1] as illustrated in Figure 1. This does not matter too much for Newtonian liquids since both shear stress and shear rate are linearly related, and hence viscosity is constant, however, for non-Newtonian liquids the shear stress has a non-linear dependence and the viscosity will thus vary at different radial locations. This can be largely corrected for by applying a non-linear correction based on the local power law index n [1] or by calculating the viscosity at  $\frac{3}{4}$  of the plate radius instead of the edge, since the shear rate for non-Newtonian and Newtonian materials have comparable values close to this point [2]. The working equations for parallel plates are given below (Equations 3 and 4) based on shear rate calculation at the plate edge and with non-linear corrections applied for shear stress. To implement the single point correction, one just assumes a Newtonian response (n = 1) with the resultant values or equations for  $\dot{\gamma}_R$  and  $\sigma$  multiplied by a factor of  $\frac{3}{4}$ .

(3)

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(2)

 $\dot{\gamma}_{R} = \frac{R\omega}{h}$ 

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Figure 4: Flow curves measured with Mixer A and a coneplate geometry on a body lotion (above) and body wash (below).

fluid, so again similar values. Equilibrium flow curves for the two non-Newtonian fluids measured with Mixer B and the cone and plate geometry are shown in Figure 5. As with Mixer A the agreement between the two configurations is very good particularly for the body lotion. For the body wash product the same discrepancy in the transition region observed with Mixer A is observed which is again attributed to the complex and variable flow field in the vicinity of the mixer.

This study clearly demonstrates the feasibility of the approach taken to estimate stress and strain constant for non-standard measuring systems and one that is relatively quick and easy to perform. As stated in the introductory section, there are clearly benefits of being able to generate comparable rheological data to that obtained with a standard geometry configuration using non-standard measuring systems or mixers and/or nonstandard vessels. This may be to replicate mixing, for keeping a sample dispersed during a measurement or to measure some rheological property that would be difficult or even impossible with a standard measurement configuration. In theory it should also be possible to generate  $C_1$  and  $C_2$  constants for larger scale mixing vessels using this approach if mixer torques and mixing speeds are known. If the power input is linearly dependent on torque then it may also be possible to replace the torque with power input in Equation 5.



Figure 5: Flow curves measured with Mixer B and a coneplate geometry on a body lotion (above) and body wash (below).

### 4 CONCLUSION

A simple and novel empirical method for determining strain/strain rate  $C_1$  and stress  $C_2$  constants for non-standard measuring systems on a rotational rheometer is proposed. This method uses relative torque measurements made with a Newtonian and non-Newtonian material and their corresponding power law fitting parameters to determine  $C_1$  and  $C_2$  using a non-linear regression analysis. Equilibrium flow curves generated for two non-Newtonian fluids using two non-standard mixing geometries showed very good agreement with data generated using a standard cone and plate configuration, therefore, validating the approach.

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