RHEOMETRY OF NON-NEWTONIAN POLYMER SOLUTION USING MICROCHANNEL PRESSURE DRIVEN FLOW

H. C. H. BANDULASENA[1], W. B. ZIMMERMAN[1], AND J. M. REES[2]

[1] University of Sheffield, Department of Chemical and Process Engineering,
Newcastle Street, Sheffield S1 3JD, U.K.
[2] Department of Applied Mathematics,University of Sheffield,
Hicks Building, Hounsfield Road, Sheffield S3 7RH, U.K.

Email: j,rees@sheffield.ac.uk
Fax: x44.114.2223809

Received: 14.11.2009, Final version: 17.5.2010

ABSTRACT:
This paper is concerned with the determination of the constitutive parameters of low concentrations of the complex fluid polyethylene glycol (PEO). Velocity fields of PEO solutions in a microfluidic T-junction have been measured for pressure driven flow using micron resolution particle image velocimetry. As the fluid is forced to turn the corner of the T-junction a range of shear rates, and therefore viscosities, is produced. Thus it is possible to establish the rheological profile from a single experiment. An inverse method used in conjunction with a finite element model was used to determine the constitutive parameters of the fluid, estimated to within 1.5 % error in all three cases considered.

ZUSAMMENFASSUNG:
Dieser Artikel befasst sich mit der Bestimmung der Konstitutivparameter von niedrigkonzentrierten Lösungen des komplexen Fluids Polyethylenglycol (PEG). Geschwindigkeitsfelder der PEG-Lösungen in einer Mikrofluidik-T-Verzweigung wurden in Druckströmungen mittels einer mikrometerauflösenden sogenannten Particle Image Velocimetry gemessen. In der T-Verzweigung muss das Fluid eine 90°-Verzweigung durchströmen, was einen breiten Schergeschwindigkeits- und Viskositätsbereich verursacht. Daher ist es auf diese Weise möglich, das rheologische Profil mit Hilfe eines einzigen Experiments zu bestimmen. Eine inverse Methode in Verbindung mit einem Finite-Elemente-Modell wird angewandt, um die Konstitutivparameter des Fluides innerhalb einer Fehlerbreite von 1.5 % zu ermitteln.

RÉSUMÉ:
Cet article traite de la détermination des paramètres constitutifs à faible concentration du fluide complexe de polyéthylène glycol (PEO). Les champs de vitesse des solutions de PEO dans une jonction T microfluidique ont été mesurés pour un écoulement en pression contrôlée à l'aide de la vélocimétrie par imagerie de particules à une résolution microscopique. Lorsque le fluide est forcé à passer le coin de la jonction T, une gamme de vitesses de cisaillement, et donc de viscosités, est générée. Ainsi il est possible d'établir un profile rhéologique à partir d'une seule expérience. Une méthode inverse est utilisée en conjonction avec un modèle d'éléments finis afin de déterminer les paramètres constitutifs du fluide avec une précision estimée d'environ 1.5 % pour les trois cas considérés.

KEY WORDS: microfluidics, rheometry, non-Newtonian, inverse problems, pressure driven flow

1 INTRODUCTION

Advances in manufacturing techniques over recent years have led to the reliable construction of miniaturized fluidic systems having dimensions in the range of 1 nm to 1 mm. One of the key advantages of this miniaturization is that chemical reactions and analyses can be performed using very small volumes of fluids, ranging from picolitres to microlitres. Response times for such systems are typically fast compared to those obtained from macro scale processing systems. The surface to volume ratio for fluid flow at these micro scales is large. This enables precise control over mass transfer, heat transfer and reaction processes [1, 2].

The characterization of fluid flow in biotechnology and chemical processing is important for fluid control, measurement and identification [3–5]. Suspensions of biocolloids and polymers often exhibit non-Newtonian behavior, such as shear dependence of viscosity, viscoelasticity and extensional viscosity. For example, at high shear rates the stretching of solid polymer molecules in solutions of xanthan gum, induces a decrease in

© Appl. Rheol. 20 (2010) 55608 DOI: 10.3933/ApplRheol-20-55608

Figure 7 (left): Graph of standard deviation σ of velocity in the region of maximum sensitivity vs. flow behavior index n.

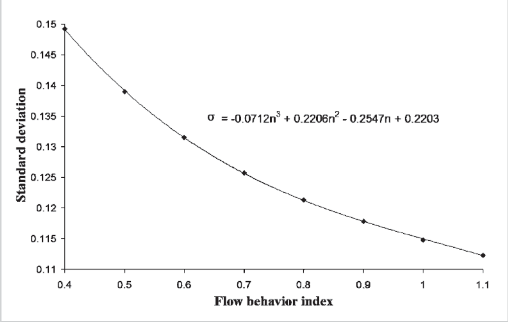
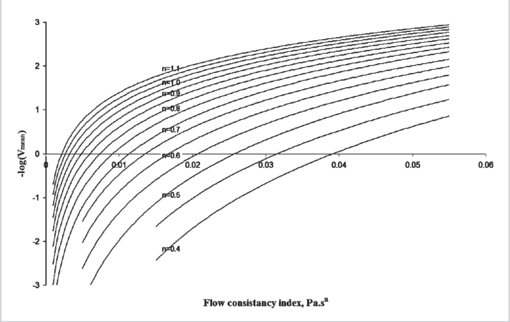


Figure 8: Graph of - log(V_{mean}) in the region of maximum sensitivity at various flow behavior indices n vs. flow consistency index K.



solution of a triangular matrix equation by back substitution.

Step 1: Find the standard deviation $\sigma_{measured}$ from statistically averaging the velocity field magnitude (second moment) over region *A*. From the results of the 8 simulations plotted in Figure 7 and the curve fit of the prediction of $\sigma(n)$, compute the root *n* of $\sigma_{measured} = \sigma(n)$. Use this root *n* in step 2.

Step 2: Find the mean velocity magnitude $V_{mean, measured}$ from statistical averaging of region *A*. From the model prediction of $V_{mean}(K, n)$, solve for the root *K* of $V_{mean, measured} = V_{mean}(K, n)$ from interpolation functions in the tabulated dataset, or equivalently, read from the graph of the family of constant *K* curves for $V_{mean}(K, n)$ predicted by the model in Figure 8.

The question for this inverse method is how good is the fit from a single microchannel μPIV estimated velocity profile of one flow experiment to the independently measured rheology of Figure 1. Table 3 shows that the power law inversion yields flow behavior indices *n* that are near unity, in which case the flow is practically Newtonian. Therefore, the flow consistency index *K* is numerically equivalent to the Newtonian viscosity μ as $n \longrightarrow 1$, which can be estimated as the zero shear rate viscosity *b* from Figure 1. Table 2 shows the discrepancy of less than 1.5% between these two quantities, well within experimental error, showing that the viscosity is nearly Newtonian in this regime of shear rates and accurately predicts its value.

It should be apparent that to capture the high shear rate power law dependence of PEO solutions shown in Figure 1, experiments with much higher (estimated as 100-fold higher) pressure drop are required. We chose the experimental conditions to match those of our earlier xanthan gum study, where xanthan gum exhibited the same low shear and high shear power law behavior. This study of PEO indicates that higher shear rates are necessary to find its regime of power law response. Only once the rheology is known either by the inverse method or conventional viscometry can the operating regime be self-consistently determined for an inverse methodology.

Table 3: Inferred values of power law flow behavior index n, $\mu_{n\longrightarrow 1}$, and the extrapolated Newtonian viscosity from the measured rheology of Figure 1. The percentage difference of the latter two numbers is given in the final column, indicating good agreement between inferred and measured constitutive relations.

Test	M_W	n	m_{n-1} [Pa·s]	b [Pa·s]	% difference
1	$1 \cdot 10^6$	1.05	0.042	0.041	1.46
2	$1 \cdot 10^6$	1.06	0.001	0.001	1.03
3	$4 \cdot 10^5$	1.05	0.055	0.055	1.28

4 CONCLUSIONS

An inverse methodology developed with a particular test system (xanthan gum), based on microchannel power law nonlinear viscosities, was applied to another, supposed power law system. The model prediction found that a self consistent inverse method could be applied to determine the power law parameters in a low shear regime with good accuracy. In the regime tested, the rheological response was nearly Newtonian, indicative that much higher pressure drops are needed to characterize the constitutive relation for this power law fluid.

ACKNOWLEDGEMENTS

HCHB would like to thank the University of Sheffield for a doctoral scholarship. WZ would like to acknowledge support from the EPSRC Basic Technology programme (EP/D004748, GR/S83746) and the Royal Academy of Engineering Senior Research Fellowship programme. We thank the ICI Polymer Centre at the University of Sheffield for the use of rheometry facilities. We acknowledge support from the EPSRC grant EP/E01867X/1 (Bridging the Gap between Mathematics, ICT and Engineering Research at Sheffield).

REFERENCES

[1] Yeo LY, Prakash-Jagadeeshan R, Friend JR: Complex Fluids and Microfluidics Workshop 2008 (CFMW08), Appl. Rheol. 19 (2009) 44 – 46.

[2] Yeo L, Chang H, Wen W: Advances in Microfluidics and Nanofluidics (Hong Kong, 2009), Appl. Rheol. 19 (2009) 175 – 176.

[3] Schweizer T: Structure and Rheology of Molten Polymers (J.M. Dealy, R.G. Larson), Appl. Rheol. 17 (2007) 258 – 259.

This is an extract of the complete reprint-pdf, available at the Applied Rheology website
http://www.appliedrheology.org

[4] Ng TS, McKinley GH, Padmanabhan M: Linear to Non-linear Rheology of Wheat Flour Dough, Appl. Rheol. 16 (2006) 265 – 274.

[5] Shibeshi SS, Collins WE: The Rheology of Blood Flow in a Branched Arterial System, Appl. Rheol. 15 (2005) 398 – 405.

[6] Guillot P, Panizza P, Salmon J B, Joanicot M, Colin A, Bruneau CH, Colin T: Viscosimeter on a Microfluidic Chip. Langmuir 22 (2006) 6438 – 6445.

[7] Zimmerman WB, Rees JM, Craven TJ: The Rheometry of Non-Newtonian Electrokinetic Flow in a Microchannel T-junction. Microfluidics and Nanofluidics 2 (2006) 481 – 492.

[8] Lee J, Tripathi A: Intrinsic Viscosity of Polymers and Biopolymers Measured by Microchip. Anal. Chem. 77 (2005) 7137 – 7147.

[9] Degre G, Joseph P, Tabeling P, Lerouge S, Cloitre M, Ajdari A: Rheology of Complex Fluids by Particle Image Velocimetry in Microchannels. Appl. Phys. Lett. 89 (2006) 024104.

[10] Bandulasena HCH, Zimmerman WB, Rees JM: An Inverse Methodology for the Rheology of a Power-Law Non-Newtonian Fluid. J. Mech. Eng. Sci. 222 (2008) 761 – 768.

[11] Bandulasena HCH, Zimmerman WB, Rees JM: Microfluidic Rheometry of a Polymer Solution by Micron Resolution Particle Image Velocimetry: A Model Validation Study. Meas. Sci. Technol. 20 (2009) 115405.

[12] Bandulasena HCH, Zimmerman WB, Rees JM: An Inverse Method for Rheometry of Power Law Fluids. J. Rheol. (2010) submitted.

[13] Bandulasena HCH, Zimmerman WB, Rees JM: Creeping flow analysis of an integrated microfluidic device for rheometry. J. Non-Newtonian Fluid Mech. 165 (2010) 1302 – 1308.

[14] Meinhart CD, Wereley ST, Santiago JG: PIV Measurements of a Microchannel Flow. Exp. Fluids 27 (1999) 414 – 419.

[15] Yu DM, Amidon GL, Weiner ND, Goldberg AH: Viscoelastic Properties of Poly(ethylene oxide) Solution. J. Pharmaceutical Sci. 83 (1994) 1443 – 1449.

[16] Di Colo G, Burgalassi S, Chetoni P, Fiaschi MP, Zambito Y, Saettone MF: Relevance of Polymer Molecular Weight to the In Vitro/In Vivo Performances of Ocular Inserts Based on Poly(ethylene oxide). Inter. J. Pharmaceutics 220 (2001) 169 – 177.

[17] Kulicke WM, Elsabee M, Eisenbach CD, Peuscher M: The Effect of Molecular Weight and Molecular Weight Distribution on the Rheological Properties of Aqueous Poly(ethylene oxide) Solution. Polymer Bulletin 9 (1983) 190 – 197.

[18] MacInnes JM, Du X, Allen RWK: Prediction of Electrokinetic and Pressure Flow in a Microchannel T-junction. Phys. Fluids 15 (2003) 1992 – 2005.

[19] Anderssen B: Inverse Problems and Emerging Techniques in Materials Characterization, Appl. Rheol. 10 (2000) 194 – 195.

[20] Hadamard J: Lectures on the Cauchy Problem in Linear Partial Differential Equations. Yale University Press, New Haven (1923).

This is an extract of the complete reprint-pdf, available at the Applied Rheology website
http://www.appliedrheology.org