Wall Shear Rates in Circular Couette Flow of a Herschel-Bulkley Fluid

Maria Chatzimina[1], Georgios Georgiou[1]* and Andreas Alexandrou[2]

[1]Department of Mathematics and Statistics, University of Cyprus, and
[2]Department of Mechanical and Manufacturing Engineering, University of Cyprus,
P.O. Box 20537, 1678 Nicosia, Cyprus

* Email: georgios@ucy.ac.cy
Fax: x357.22892601

Received: 30.9.2008, Final version: 12.1.2009

Abstract:
The objective of this work is to study quantitatively the errors introduced by the standard Newtonian and power-law assumptions used in the determination of the material properties of viscoplastic fluids from circular Couette experiments. The steady-state circular Couette flow of a Herschel-Bulkley fluid is solved assuming that the inner cylinder is rotating at constant speed while the outer one is fixed. Analytical solutions are presented for certain values of the power-law exponent. It is shown that the error in the computed wall shear rate, which is insignificant when the diameter ratio is closed to unity, may grow large depending on the diameter ratio and the material parameters.

Zusammenfassung:
Ziel dieser Arbeit ist eine quantitative Untersuchung der Fehler, die in die Ermittlung der Materialeigenschaften viscoplastischer Flüssigkeiten in Rotations-Couette Experimenten durch die üblichen Newtonschen und Potenzgesetzannahmen eingeführt werden. Der stationäre Rotations-Couette-Fluss einer Herschel-Bulkley Flüssigkeit wird gelöst unter der Annahme, dass der innere Zylinder sich mit konstanter Geschwindigkeit dreht, während der Äussere ruht. Für bestimmte Werte des Exponenten im Potenzgesetz werden analytische Lösungen angegeben. Es wird gezeigt, dass der bei Durchmesserverhältnissen nahe Eins bedeutungslose Fehler in der berechneten Wandscherrate in Abhängigkeit vom Durchmesserverhältnis und den Materialkonstanten große Werte annehmen kann.

Résumé:
L'objectif de ce travail est d'étudier quantitativement les erreurs présentées par les suppositions ordinaires newtoniennes et de loi de puissance faites pour la détermination des propriétés matérielles des fluides viscoplastiques des expériences circulaires de Couette. L'écoulement permanent de Couette circulaire d'un fluide de Herschel-Bulkley est résolu supposant que le cylindre intérieur tourne à la vitesse constante tandis que le cylindre extérieur est fixe. Des solutions analytiques sont présentées pour certaines valeurs de l'indice de loi de puissance. On montre que l'erreur du taux de cisaillement calculé au paroi, qui est insignifiant quand le rapport de diamètre est proche de l'unité, peut se développer grande selon le rapport de diamètre et les paramètres matériels.

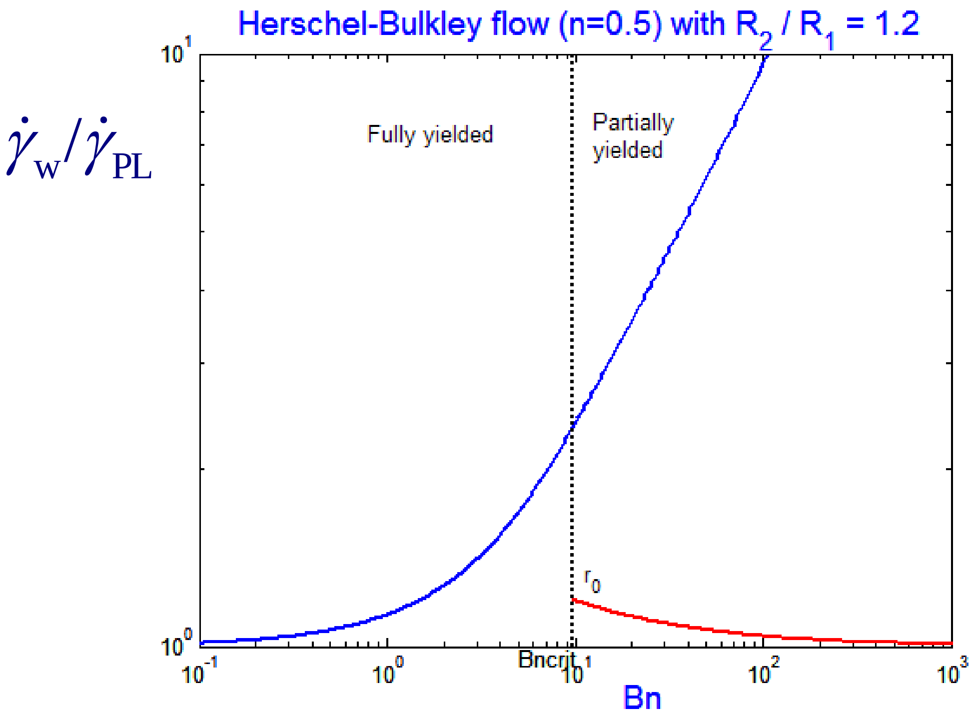
Key words: circular Couette flow, Herschel-Bulkley fluid, Bingham plastic, viscometry

1 INTRODUCTION

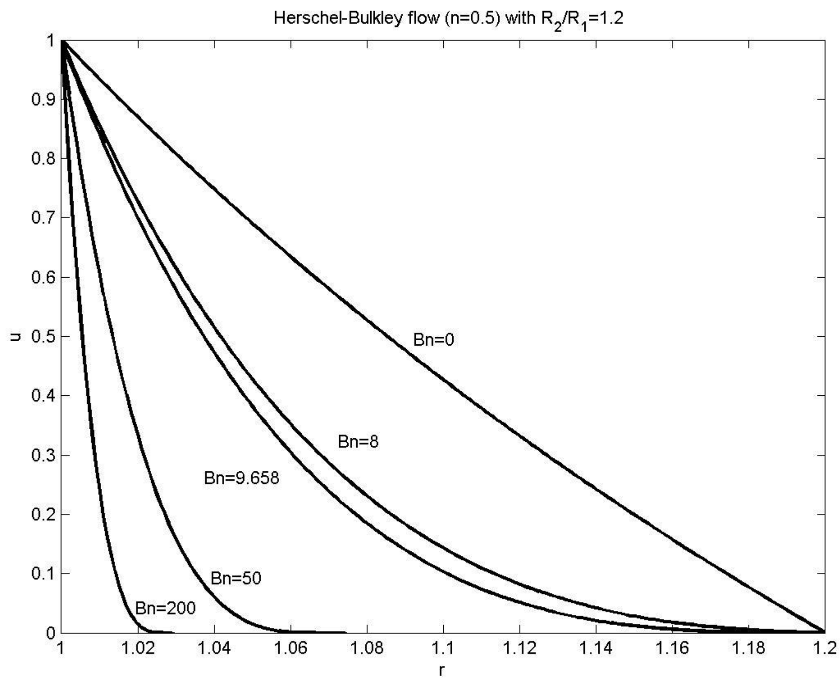
Circular Couette flow is a standard, widely used viscometric flow for many fluids including materials with yield stress, such as paints, cosmetics, drilling fluids and aqueous bentonite suspensions [1], liquid foods [2, 3], granular suspensions [4], cement paste and fresh concrete [5], and semisolid metal slurries [6]. An interesting article discussing the origins of Couette rheometry has been recently published by Dontula et al. [7]. Macosko [8] discusses the underlying assumptions as well as the possible sources of error, such as end effects, wall slip, eccentricities, and viscous heating.

Converting torque measurements in a Couette rheometer to flow curve data, i.e. to the shear stress versus shear rate plot, is an ill-posed inverse problem, known as Couette inverse problem, which becomes more complicated in the case of fluids with yield stress, due to the possible existence of an unsheared region at the fixed cylinder [9]. In the past three decades, the issue of the correct determination of the shear rate has been addressed by various investigators, who proposed some procedures to overcome certain limitations. A nice review including early works on the subject is provided by Estellé et al. [3]. It should be noted that for the characterization of

This is an extract of the complete reprint-pdf, available at the Applied Rheology website
http://www.appliedrheology.org



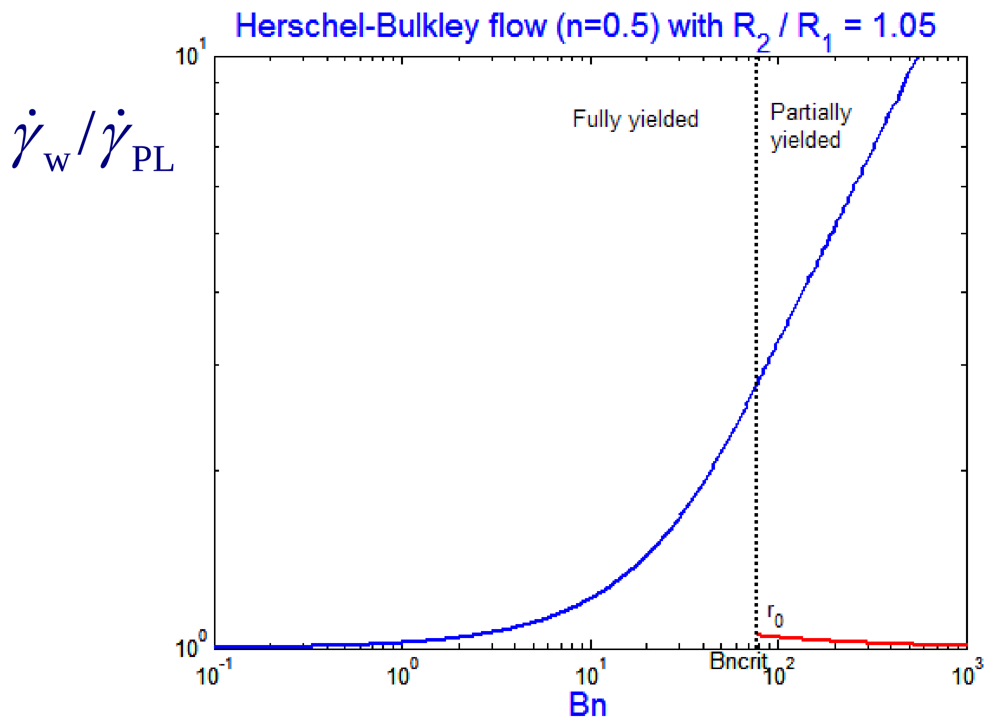
(a)



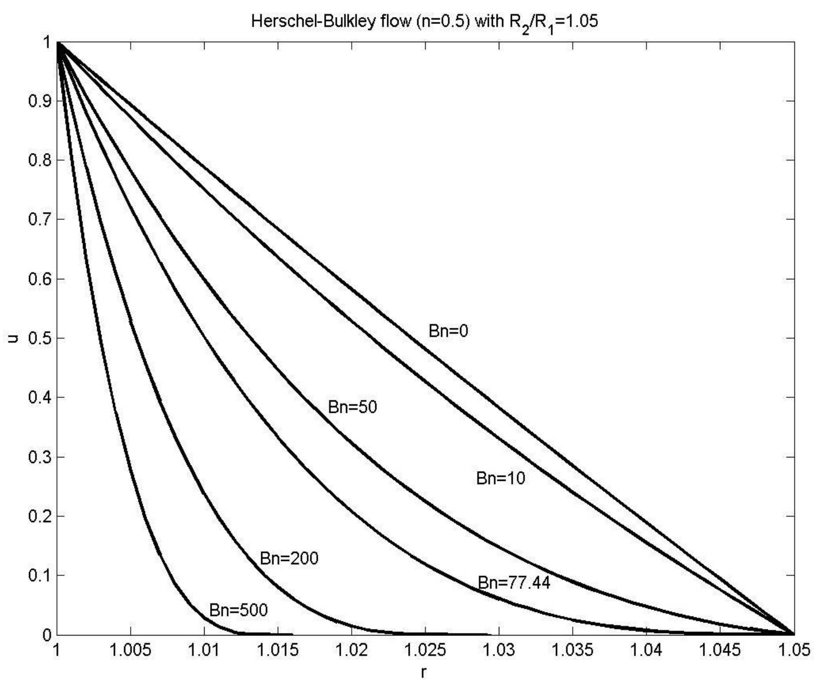
(b)

Figure 6. Results for Herschel-Bulkley flow (n=0.5) with R_2/R_1=1.2: (a) Wall shear rate ratio, $\dot{\gamma}_w / \dot{\gamma}_{PL}$, calculated from Eqs. (53) and (33). The dotted line shows the critical Bingham (Bn^{crit}=9.658) number above which the fluid is partially yielded. (b) Velocity profiles in both the fully- and partially-yielded regimes.

This is an extract of the complete reprint-pdf, available at the Applied Rheology website
http://www.appliedrheology.org



(a)



(b)

Figure 7. Results for Herschel-Bulkley flow (n=0.5) with R_2/R_1=1.05: (a) Wall shear rate ratio, $\dot{\gamma}_w / \dot{\gamma}_{PL}$, calculated from Eqs. (53) and (33). The dotted line shows the critical Bingham (Bn^{crit}=77.45) number above which the fluid is partially yielded. (b) Velocity profiles in both the fully- and partially-yielded regimes.

This is an extract of the complete reprint-pdf, available at the Applied Rheology website
http://www.appliedrheology.org