Simple and Accurate Formulas for Flow-Curve Recovery from Couette Rheometer Data

F. R. de Hoog and R. S. Anderssen*

CSIRO Mathematical and Information Sciences, GPO Box 664, Canberra ACT 2601, Australia

*E-mail: bob.anderssen@csiro.au
Fax: +61.2.6216 7111

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Abstract:
In Couette rheometry, most of the current flow-curve recovery algorithms require the explicit numerical differentiation of the measured angular velocity data. The exceptions and popular choices, because it avoids the need for a numerical differentiation, are the parallel plate approximation (cf. Bird et al. [1], Table 10.2-1) and the simplest of the formulas given in Krieger and Elrod [2]. However, their applicability is limited to narrow gap rheometer data. In this paper, equally simple formulas are presented which are exact for Newtonian fluids, do not involve a numerical differentiation and are consistently more accurate than the simple formulas mentioned above. They are based on a generalization of the Euler-Maclaurin sum formula solution of the Couette viscometry equation given in Krieger and Elrod. As well as illustrating the improved accuracy for the recovery of flow-curves for fluids with and without a yield-stress, details about more general and accurate formulas for flow-curve recovery from Couette rheometry data are given. The situation for the recovery of flow-curves from wide gap rheometery measurements is also discussed.

Zusammenfassung:
Bei der Couette-Rheometrie erfordern die meisten gegenwärtigen Fliesskurven-Bestimmungsalgorithmen die explizite numerische Differentiation der gemessenen Drehgeschwindigkeitsdaten. Die Ausnahmen und beliebte Wahl sind die parallelen Platten-Approximation (Bird et al. (1977) Tafel 10.2-1) und die einfachste der Formeln in Krieger und Elrod (1953), da sie die numerische Differentiation vermeiden. Jedoch ist ihre Anwendbarkeit auf Rheometerdaten mit einem engen Spalt beschränkt. In diesem Artikel werden ebenfalls einfache Formeln dargestellt, die für Newtonsche Fluide exakt sind, jedoch keine numerische Differentiation verwenden und genauer sind als die oben angegebenen einfachen Formeln. Sie basieren auf einer Verallgemeinerung der Euler-Maclaurin-Summenformeln-Lösungen der Couette-Viskometrie-Gleichung in Krieger and Elrod. Details über die allgemeineren und genaueren Formeln für die Fliesskuven von der Couette-Rheometrie werden gegeben. Um die verbesserte Genauigkeit für die Bestimmung der Fliesskurven für Fluide mit und ohne Fliesspannung gleichermassen zu illustrieren, werden Details über allgemeinere und akkuratere Formeln für die Fliesskurven-Bestimmung aus Couette-Rheometrie-Daten gegeben. Die Situation für die Bestimmung der Fliesskurven aus rheometrischen Messungen mit einem grossen Spalt wird auch diskutiert.

Résumé:
Dans les écoulements de type "Couette", la plupart des algorithmes visant à obtenir des lignes de flux exigent de différencier numériquement les vitesses angulaire mesurées. Font exception à cette règle et de fait populaires sont les approximations de type "plaques parrallèles" (Bird et al. (1977) Table 10.2-1) et aussi, la plus simple des formules donnée par Krieger et Elrod (1953). Pourtant, leur domaine de validité est limité aux données produites par des rhéomètres à espacement étroit. Dans cette contribution, nous présentons des formules simples, exactes pour les fluids newtoniens, n'exigeant pas de differentiation numérique, et qui sont plus précises et simples que les formules mentionées plus haut. Elles sont basées sur une généralisation de la formule de sommation d'Euler-Maclaurin pour l'équation viscométrique produite par Krieger et Elrod. Nous montrons que la précision des lignes de flux pour des fluides avec et sans limite élastique est améliorée et fournissons les détails concernant des formules exactes et générales pour la détermination des lignes de flux en géometrie de type Couette. La situation concernant la détermination des lignes de flux dans les rhéometres à large espacement est aussi discutée.

Key words: Flow-curve recovery, Couette rheometer, Euler-MacLaurin sum formula, non-Newtonian, finite difference formulas, Williamson, power law, Casson

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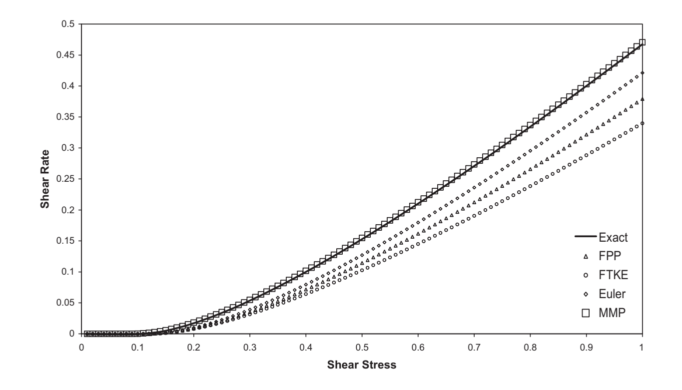

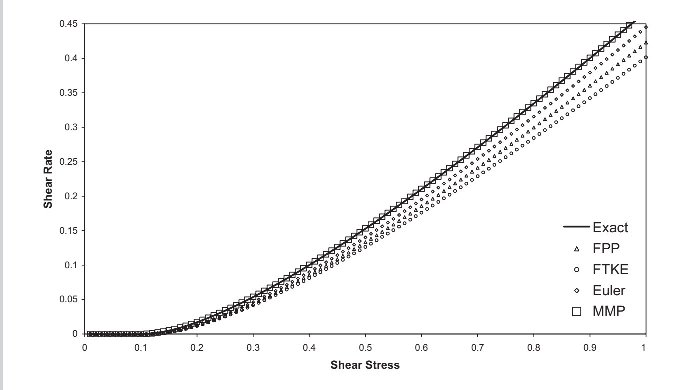

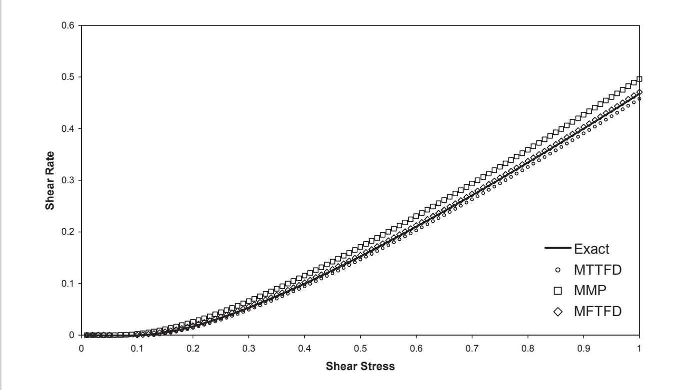

by the presence of measurement errors. When the errors are substantial, it will be necessary to first smooth the data (Ancey [26]) before applying the approximations discussed above.

This descriptive assessment of the effect of observational error can be illustrated in the following manner. Let ϵ denote an error with mean zero and variance var [ϵ], and assume that the effect of the observational error has a relative effect in the sense that the estimates of $\dot{\gamma}_{FPP}(\sigma)$ and $\dot{\gamma}_{MMP}(\sigma)$ take, respectively, the form

$$\hat{\dot{\gamma}}_{FPP}(\sigma)=\frac{\Omega(\sigma)(1+\epsilon)}{(1-s)}$$

and

$$\hat{\dot{\gamma}}_{MMP}(\sigma)=\frac{2s\Omega(\sigma s^{-1})(1+\epsilon)}{(1-s^2)}$$

A simple calculation then shows that

$$\mathrm{var}\left[\hat{\dot{\gamma}}_{FPP}(\sigma)\right]=\frac{\Omega^2(\sigma)\mathrm{var}[\epsilon]}{(1-s)^2}$$

and

$$\mathrm{var}\left[\hat{\dot{\gamma}}_{MMP}(\sigma)\right]=\left(\frac{2s}{(1-s^2)}\right)^2\Omega^2(\sigma s^{-1})\mathrm{var}[\epsilon]$$

This establishes that relative errors in the measurement of $\Omega(\sigma)$ have a relative error effect on the estimation of the different estimates of the shear rate. Since $2s/(1+s)<1$, it follows that the multiplier in var [$\dot{\gamma}_{MMP}(\sigma)$] is smaller than that in var[$\dot{\gamma}_{FPP}(\sigma)$].

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Figure 6 (left above): The recovery of the flow curve (shear rate as a function of shear stress) for a Casson fluid by the FPP, FTKE, Euler and MMP approximations from exact angular velocity data for a Couette rheometer with s = 0.8.

Figure 7 (right above): The recovery of the flow curve (shear rate as a function of shear stress) for a Casson fluid by the FPP, FTKE, Euler and MMP approximations from exact angular velocity data for a Couette rheometer with s = 0.9.

Figure 8 (below): The recovery of the flow curve (shear rate as a function of shear stress) for a Casson fluid by the MMP, MTTFD and MFTFD approximations from exact angular velocity data for a Couette rheometer with s = 0.5.

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