# Elongational Viscosity Measurements Using a Semi-Hyperbolic Die

D. G. Baird* and J. Huang

Department of Chemical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

* E-mail: dbaird@vt.edu
Fax: x1.540.231.2732

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**Abstract:**
The lubricated semi-hyperbolic die has been proposed as a technique for generating uni-axial extensional flow and, hence, as a device for measuring elongational viscosity. Two methods for extracting extensional viscosity data for polymer melts in laminar flow from this device have been proposed and are evaluated here. Following the approach proposed by Collier and coworkers, values of the transient extensional viscosity, $\eta_e^+$, obtained from a non-lubricated semi-hyperbolic (SHPB) die for several polyethylene (PE) melts were found to be considerably higher than values obtained by means of the Münstedt-type device. Furthermore, the values of $\eta_e^+$ obtained from the SHPB die were considerably higher than the strain averaged values of $\eta_e^+$ which Everage and Ballman proposed would be obtained from a lubricated SHPB. The pressure drop across a SHPB die was estimated assuming resistance was all due to wall shear (using the lubrication approximation) for two PE resins. In the case of low density PE (LDPE) the values agreed to within 20% of the measured values suggesting that shear effects at the die wall were dominating the pressure drop and not extensional stresses. An analysis was carried out which showed that in the presence of lubrication the conditions for which the values of $\eta_e^+$ obtained from the SHPB would be relatively accurate (Hencky strains > 5.0).

**Zusammenfassung:**
Die geschmierte semi-hyperbolische Düse wurde als Methode vorgeschlagen, um uniaxiale Dehnströmungen zu erzeugen und somit als Gerät zur Messung der Dehnviskosität zu dienen. Zwei Methoden, um Dehnviskositätsdaten für Polymerschmelzen in laminarer Strömung mit Hilfe dieser Technik zu extrahieren, wurden vorgeschlagen und werden in diesem Artikel evaluiert. Die Werte der transienten Dehnviskosität $\eta_e^+$ von verschiedenen Polyethylenen (PE), die mittels des Ansatzes von Collier und Mitarbeitern aus einer ungeschmierten semi-hyperbolischen Düse (SHPB) erhalten wurden, waren wesentlich höher als die Werte, die mittels des Münstedtrheometers gemessen wurden. Darüber hinaus waren die Werte von $\eta_e^+$, die mittels der SHPB erhalten wurden, wesentlich höher als die dehnungsgemittelten Werte von $\eta_e^+$, die nach Everage and Ballman mittels der geschmierten SHPB erhalten wurden. Der Druckabfall innerhalb einer SHPB-Düse wurde für zwei Polyethylene abgeschätzt unter der Annahme, dass der Widerstand ausschliesslich von der Wandscherung resultierte (mit Hilfe der Schmierfilmnäherung). Für das PE mit niedriger Dichte (LDPE) stimmten die Werte innerhalb 20 % mit den gemessenen Werten überein, was vermuten lässt, dass Schereffekte an der Düsenwand den Druckabfall dominierten und nicht die Dehnspannungen. Eine Analyse wurde durchgeführt, die zeigte, dass bei Schmierung die Bedingungen relativ genau sind (Hencky-Dehnungen > 5.0), die für $\eta_e^+$ mit Hilfe der SHPB erhalten wurden.

**Résumé:**
La Filière semi-hyperbolique avec lubrication a été proposée comme technique pouvant générer un écoulement extensionnel uni axial, et donc, comme un appareil pour mesurer la viscosité d'élongation. Ici, nous proposons et évaluons deux méthodes afin d'extraire des données de viscosité d'élongation à partir de cet appareil pour des fondus de polymère en écoulement laminaire. En suivant l'approche proposée par Collier et al., les valeurs de la viscosité d'élongation transitoire, $\eta_e^+$, obtenues avec une filière semi-hyperbolique non lubriquée (SHPB) pour plusieurs fondus de polyéthylène, se sont avérées être considérablement plus grandes que les valeurs obtenues au moyen de l'appareil de type Münstedt. De plus, les valeurs de $\eta_e^+$ obtenues avec la filière SHPB sont considérablement supérieures aux valeurs moyennées sur la déformation, ce que Everage et Ballman ont envisagé obtenir avec une SHPB lubriquée. La chute de pression dans la filière SHPB a été estimée en supposant que la résistance est due au cisaillement aux parois (en utilisant l'approximation de lubrication) pour deux résines de PE. Dans le cas du PE de basse densité (LDPE) les valeurs sont en accord avec celles mesurées dans une marge de 20%, ce qui suggère que les effets de cisaillement aux parois de la filière dominent la chute de pression et non les contraintes extensionnelles. Une analyse a été entreprise qui montre que avec la présence de lubrication, les conditions pour lesquelles les valeurs de $\eta_e^+$ obtenues avec la SHPB seraient relativement précises (déformations de Hencky > 5.0).

**Key words:** elongational viscosity, semi-hyperbolic die, extensional rheometry, polyethylene





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This is the desired transient normal stress difference needed to obtain the time or strain averaged elongational viscosity at any $z$ position (which corresponds to time or strain). We note that at $z = z_e$, $P(R) = 0$ and, hence, $\pi_{zz} = -\Delta P_o$ which is consistent with Everage and Ballman's result. Hence, it seems that it may be possible to obtain $\eta_e^+$ at any instant or strain for large strains (in the case we analyzed here for strains greater than about 5.0) provided the pressure at various $z$ positions is measured along the wall of the die (this should feasible by using pressure taps and placing the transducers along a spiral path around the die). We propose that Eq. 12 is the basis for building an extensional rheometer which should give values of $\eta_e^+$ at least at high strains. We note that James and coworkers [9 - 10] built a device with pressure transducers as suggested here for measurements of extensional viscosity at high *Re*, but did not realize the correction required for geometric considerations and that the values would be time or strain averaged values.

Finally, we note that this solution is based on the assumption of slip at the die wall (i.e. lubrication). There still exists the possibility that a significant contribution to the pressure comes from viscous shear effects at the die wall. The die profile after about the first 15 % of the length, $z_e$, is of only slightly varying radius (Fig. 12a), and the capillary walls are expected to contribute significantly to the resistance of flow. Although it may be possible to build in a correction to Eq. 12, this aspect must be more fully evaluated before we can expect to obtain reliable values of elongational viscosity. Certainly a full numerical simulation of the die is necessary in the future in order to assess the degree of slip required to minimize wall shear effects and to evaluate the assumptions which allow one to eliminate the geometrical contributions to the stresses (i.e. $\pi_{rz}$). Feigl and coworkers [14] have initiated this type of study using a K-BKZ integral constitutive relation and did report that at Hencky strains of 6 and 7 in the presence of complete slip that values of extensional viscosity were obtained which agreed with those determined by integrating the constitutive equation over the same range of strain or time. However, a closer look at their results shows that there is significant disagreement between the numerically calculated values and those obtained by integrating the constitutive equation over time (or strain). It seems that a thorough numerical solution could allow one to determine the degree of slip required to get reasonably accurate values of the extensional viscosity and the magnitude of the correction for the geometric factor.

## 5 CONCLUSIONS

Values of the transient extensional viscosity, $\eta_e^+$, obtained from a semi-hyperbolic die without lubrication following the method proposed by Collier and coworkers for several polyethylene melts were significantly higher than values obtained from a Münstedt device and $3\eta_o$ over a range of extension rates from 0.01 to 1.0 $s^{-1}$. Furthermore, the values of $\eta_e$ obtained from the SHPB die were considerably higher than the strain averaged values of $\eta_e^+$ which Everage and Ballman proposed would be obtained from a lubricated SHPB. Based on calculations of the pressure drop across a SHPB die designed to produce a Hencky strain of 7.0, it is apparent that the pressure drop is dominated by shear at the die wall when no lubrication is used. In the case of LLDPE, the pressure drop calculated by means of the lubrication approximation was considerably higher than the measured values, but this was attributed to slip-stick. However, in spite of slip-stick, values of $\eta_e^+$ obtained from the SHPB die were significantly higher than the values obtained from the Münstedt device. A new analysis was presented which showed that in the presence of lubrication the values of $\eta_e^+$ (or at least strain averaged values) obtained from the SHPB will be valid at Hencky strains greater than about 5.0. This conclusion was reached based on geometric considerations and the fact that the principal stresses align with the principal strain rates in the SHPB only at high strains. Collier and coworkers have now acknowledged that the flow is unsteady in the Lagrangian sense and, hence, the stresses do not reach steady values until the high strains. In the presence of complete slip they claim, based on numerical calculations, that reasonably accurate values of extensional viscosity are obtained. However, no new analysis is proposed for extracting extensional viscosity values for pressure drop and flow rate measurements.





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