

# PHENOMENOLOGICAL APPROACH OF THE EFFECTIVE VISCOSITY OF HARD SPHERE SUSPENSIONS IN SHEAR-THINNING MEDIA

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## ABSTRACT:

In this work we investigate the rheological behaviour of macroscopic buoyant hard spheres dispersed in a shear-thinning suspending fluid. We focus on the phenomenological study of the influence of the shear-thinning behaviour of the suspending medium on the effective apparent suspension viscosity at different volume fractions. In the oil industry, the effective viscosity concept is widely used and very useful to quickly characterize a change of viscosity due to an increase of the solid content. Viscosity measurements are compared to the effective viscosity of a suspension of hard spheres in an Ostwald fluid. The power law index of the suspending fluid is shown, both experimentally and theoretically, to influence strongly the volume fraction dependence of the suspension effective viscosity. All experimental results are shown to be quite correctly plotted on a master curve, with only one adjustable parameter, the maximum packing fraction  $\phi_m$ . The best fit is obtained for  $\phi_m = 0.57$ , corresponding to the theoretical maximum random packing volume fraction.

## ZUSAMMENFASSUNG:

In dieser Arbeit untersuchen wir das rheologische Verhalten von makroskopischen, schwimmenden harten Kugeln, die in einer scherverdünnenden Flüssigkeit suspendiert sind. Der Fokus liegt auf einer phänomenologischen Untersuchung des Einflusses des scherverdünnenden Verhaltens des suspendierenden Mediums auf die effektive scheinbare Viskosität der Suspension bei verschiedenen Volumengehalten. In der Ölindustrie wird das Konzept der effektiven Viskosität häufig angewandt und ist sehr tauglich zur schnellen Charakterisierung der Viskositätsänderung aufgrund eines höheren Feststoffanteils. Viskositätsmessungen werden verglichen mit der effektiven Viskosität einer Suspension aus harten Kugeln in einer Ostwald-Flüssigkeit. Es wird sowohl theoretisch als auch experimentell gezeigt, dass der Power-Law-Index des suspendierenden Mediums die Abhängigkeit des Volumenanteils auf die effektive Viskosität der Suspension erheblich beeinflusst. Sämtliche experimentellen Resultate können in einer Masterkurve mit nur einem Fitparameter, der maximalen Packungsdichte  $\phi_m$ , dargestellt werden. Den besten Fit findet man für  $\phi_m = 0,57$ , was der theoretischen maximalen Packungsdichte bei einer zufälligen Anordnung entspricht.

## RÉSUMÉ:

Dans de travail nous étudions le comportement rhéologique de sphères dures macroscopiques flottantes, suspendues dans un fluide rhéo-amincissant. Nous nous concentrons sur l'étude phénoménologique de l'influence du comportement rhéo-amincissant du milieu suspensif sur la viscosité apparente de la suspension à diverses fractions volumiques. Dans l'industrie pétrolière, le concept de la viscosité effective est largement utilisé et est très pratique pour caractériser rapidement un changement de viscosité du à une augmentation de la fraction solide. Les mesures de viscosité sont comparées à la viscosité effective d'une suspension de sphères dures dans un fluide de type Ostwald. Il est montré expérimentalement et théoriquement que l'index de la loi de puissance du fluide suspensif influence fortement la dépendance de la viscosité effective de la suspension avec la fraction volumique. Tous les résultats expérimentaux se sont avérés être bien représentés sous la forme d'une courbe maîtresse, obtenue en ajustant un seul paramètre, la fraction de package maximum  $\phi_m$ . Le meilleur ajustement est obtenu pour  $\phi_m = 0,57$ , correspondant à la valeur théorique pour la fraction volumique maximum dans le cas d'un package aléatoire.

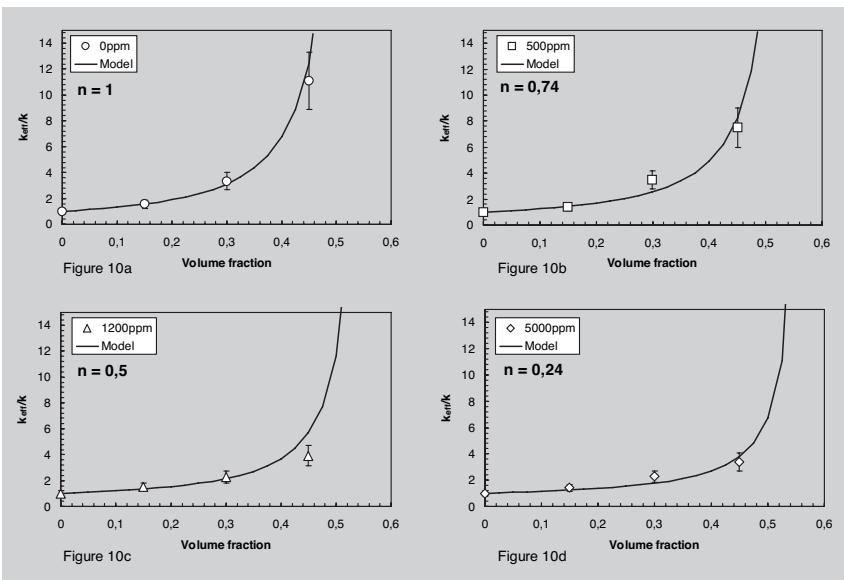
**KEY WORDS:** suspension, hard spheres, shear-thinning, power law index, effective viscosity

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**Figure 10 (left):**  
Reduced effective consistency as a function of solid volume fraction, at four power law index values.

**Figure 11:**  
Reduced effective consistency  $k_{\text{eff}}/k$  as a function of  $1-n$ .

$$\frac{k_{\text{eff}}}{k} = \frac{1-\phi}{(1-\phi/\phi_m)^{n+1}} \quad (13)$$

Figure 10 clearly shows that the effect of the effective consistency index is all the more marked as the shear-thinning behaviour is less marked. From a more quantitative point of view, it shows that  $k_{\text{eff}}/k$  increases by a factor 6 when the solid volume fraction increases from 15 to 45 % in a Newtonian suspending fluid, whereas the increase of the ratio  $k_{\text{eff}}/k$  is only 2 for the same increase of volume fraction in a high shear-thinning suspending medium, characterized by a power law index  $n = 0.24$ .

To emphasise this point and to show the strong and explicit dependence of the suspension effective viscosity with  $n$ , we plot the evolution of  $k_{\text{eff}}/k$  with  $1-n$  for different volume fractions in Fig. 11.

### 5.3 MASTER CURVE

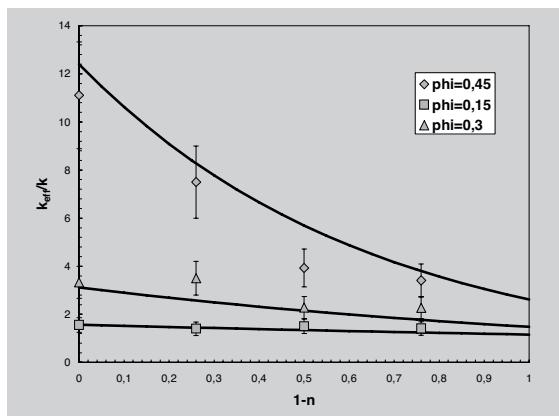
The different experimental results can be plotted on a master curve when considering the following quantity:

$$\xi = \left( \frac{k_{\text{eff}}}{(1-\phi)k} \right)^{\frac{1}{n+1}} \quad (14)$$

From Eq. 11, the dependence of  $\xi$  with the solid volume fraction should be simply:

$$\xi = 1 - \frac{\phi}{\phi_m} \quad (15)$$

Figure 11 shows indeed that  $\xi$  versus  $\phi$  dependence follows Eq. 15 quite well with only one adjustable parameter, the maximum packing



volume fraction, fixed at 0.57, which is in quite good agreement with theoretical predictions.

## 6 CONCLUSION

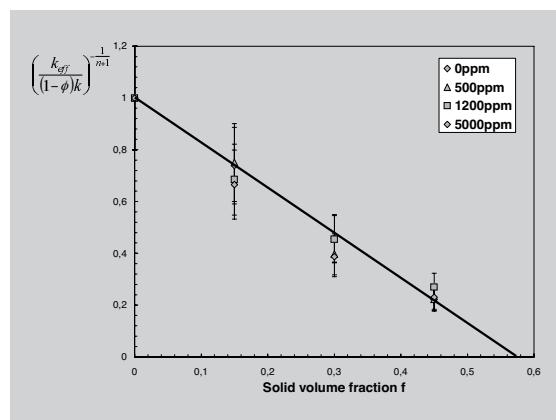
In this paper, experiments carried out on model suspensions of monodisperse buoyant hard spheres clearly show that the shear-thinning behaviour of the suspensions is governed by the shear-thinning properties of the suspending fluid. More precisely, the power law index  $n$  characterising the shear-thinning behaviour of the suspensions, over a large shear rate range, was shown to be equal to that characterising the shear-thinning behaviour of the suspending fluid, for various  $n$  values, ranging from 1 to 0.24.

The experimental results were shown to be in quite good agreement with a phenomenological analysis of the apparent effective viscosity of the suspensions, which explicitly take into account the dependence with the shear rate, the power law suspending fluid parameters (consistency and power law index), and the solid volume fraction. In particular, the model predicts that the solid volume fraction dependence of the effective viscosity is all the more marked as the non-linear behaviour is less marked, as shown by experimental results.

Using this phenomenological approach, all experimental results were plotted on a master curve, with only one adjustable parameter, the maximum packing volume fraction, the best fit was obtained for  $\phi_m = 0.57$ , corresponding to the theoretical maximum random packing volume fraction.

From an industrial point of view, the fact that the shear-thinning behaviour of a suspension can be governed by the shear-thinning properties of the suspending fluid is very valuable. Indeed, increase of apparent viscosity due to solid particles can be limited by the choice of adapted power law interstitial fluids. In the case of cutting removal by drilling mud, the amount of solids is not controlled but imposed by the rate of penetration of the drilling process. Still, as

**Figure 12:**  
Normalized consistency as a function of solid volume fraction (The straight line corresponds to Eq. 15).



drilling muds behave like shear-thinning fluids, the choice of a suspending fluid with a small power law index  $n$  could be a way to limit the increase of viscosity for drilling with large rates of penetration. Enhancing the shear-thinning behaviour of the suspending fluid can be performed by adding carefully chosen polymers at appropriate concentrations.

At last, we would like to point out that the present work was limited to the study of non-Newtonian viscous behaviour; future work will be concerned by the effect of other non-linearities, such as non-linear elastic properties.

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