CHAOTIC MIXING ANALYSES BY DISTRIBUTION MATRICES

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ABSTRACT
Distributive fluid mixing in laminar flows is studied using the concept of concentration distribution mapping matrices, which is based on the original ideas of Spencer & Wiley [1], describing the evolution of the composition of two fluids of identical viscosity with no interfacial tension. The flow domain is divided into cells, and large-scale variations in composition are tracked by following the cell-average concentrations of one fluid using the mapping method of Kruijt et al. [2]. An overview of recent results is presented here where prototype two- and three-dimensional time-periodic mixing flows are considered. Efficiency of different mixing protocols are compared and for a particular example the (possible) influence of fluid rheology on mixing is studied. Moreover, an extension of the current method including the microstructure of the mixture is illustrated. Although here the method is illustrated making use of these simple flows, more practical, industrial mixers like twin screw extruders can be studied using the same approach.

ZUSAMMENFASSUNG
Die Vermischung von Flüssigkeiten in laminarer Strömung wird anhand der "Concentration Distribution Mapping Matrice"-Methode, basierend auf den Arbeiten von Spencer & Wiley, untersucht [1]. Durch diese Methode wird die zeitliche Entwicklung der Zusammensetzung zweier Flüssigkeiten gleicher Viskosität ohne Grenzflächenspannung beschrieben. Hierfür wird der Strömungsbereich zunächst in Zellen unterteilt und anschließend wird die Veränderung der Zusammensetzung, durch das Verfolgen der zellengemittelten Konzentrationen eines der beiden Fluide mittels der "Mapping Method" nach Kruijt et al., beobachtet [2]. Dieser Beitrag gibt ein Überblick über neuere Resultate zu typischen zwei- und dreidimensionalen, zeitperiodischen Mischungsströmungen. Die Effizienz verschiedener Mischmethoden wird verglichen und für ein ausgewähltes Beispiel wird der (mögliche) Einfluss der rheologischen Eigenschaften der Flüssigkeiten auf den Mischungsvorgang untersucht. Ausserdem wird eine Erweiterung der Methode vorgestellt, die die Mikrostruktur der Mischung miteinbezieht. Obwohl diese Methode hier an einfachen Strömungssituationen illustriert wird, kann sie auch zur Untersuchung von mehr anwendungsorientierten, industriellen Mischern, wie dem Doppelwellenextruder verwendet werden.

RÉSUMÉ
Le mélange distributif de fluide dans les écoulements laminaires est étudié au moyen du concept de matrices de traçage de distributions de concentration, qui est basé sur les idées originales de Spencer & Wiley [1], décrivant l'évolution de la composition de deux fluides avec des viscosités identiques et sans tension interfaciale. Le domaine d'écoulement est divisé en cellules et les variations à grande échelle en composition sont suivies en estimant les concentrations moyennes d'un des deux fluides à l'aide de la méthode de tracé de Kruijt et al. [2]. Une revue des résultats récents est ici présentée, où des écoulements de mélange prototypes à deux et trois dimensions et avec une dépendance temporelle périodique sont considérés. Les efficacités des différents protocoles de mélange sont comparées et pour un exemple particulier, l'influence (éventuelle) de la rhéologie du fluide sur l'action de mélange est étudiée. De plus, une extension de la méthode, où la microstructure du mélange est incorporée, est présentée. Malgré le fait que nous donnions ici des exemples où la méthode est appliquée dans le cas d'écoulements simples, d'un point de vue plus pratique, des mélangeurs industriels, comme les extrudeurs à vis jumelles, peuvent être étudiés en utilisant une approche similaire.

1 INTRODUCTION

Fluid mixing processes receive a considerable amount of recognition because of their importance in nature and industry. Although in many cases mixing is associated with turbulent fluid motions, mixing of very viscous fluids constitutes an important class of mixing occurrences. These are typical for polymer blending, compounding, food processing etc, and some new models to study these flows are overviewed in this paper. It is well known that even for laminar flow at very low Reynolds numbers mixing can lead to complex flow patterns, and that the exposure of such patterns may be understood by imploring the theory of dynamical systems [3]. The motion of passive particles in such a flow is described by the set of ordinary differential equations,

$$\dot{\mathbf{x}} = \mathbf{u}(\mathbf{x}, t) \quad (1)$$

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ance, but also important fundamental information about the flow itself. The calculation reveals the self-replicating pattern of interface orientation that arises in periodic flows. When the flow is globally chaotic, this pattern displays an exponential growth rate for interfacial area. Spatial analyses of the one-period dynamics for interface stretching can be performed, to reveal which parts of the flow field are most effective at small-scale mixing.

One disadvantage of the present calculation is that it is subject to numerical diffusion. This restricts its quantitative accuracy, especially when studying the long-time behaviour of a flow. However, the short-time behaviour is accurately predicted, and we expect that the relative performance of different mixing protocols will be predicted quite accurately, provided one uses the same size mapping steps for both protocols. This makes the extended mapping method a useful engineering tool.

A topic of the current research in our laboratory, is the extension of the mapping method technique to more complex, industrial, mixers. Examples include the multiflux static mixer, described by Sluijters [32], an example of a three-dimensional space-periodic flow, and the closely intermeshing, corotating twin screw extruder, which can be regarded as either time- or space-periodic. Moreover, an experimental set-up of the cubic cavity flow will be used to validate the computational results for e.g. the four-step mixing protocol, and to study the influence of visco-elastic effects on mixing.

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BIOGRAPHY

Han Meijer received his Ph.D. degree from the University of Twente in 1980 with Prof. J.F. Ingen Housz as his supervisor. He joined DSM research, and was active in the area of Polymer Processing Modelling and Explorative Research. In 1985 he became part-time professor at the department of Polymer Chemistry and Technology in the area of applied rheology. In 1989 he became full professor in Polymer Technology in the Division of Computational and Experimental Mechanics of the Department of Mechanical Engineering. He has been chairman of the Dutch Society of Rheology since 1995 and is currently president-elect of the Polymer Processing Society. Patrick Anderson graduated in Applied Mathematics at the University of Eindhoven in 1994 under the supervision of Prof. A.A. Reusken. He received his Ph.D. degree in the research group Materials Technology lead by Prof. H.E.H. Meijer and Prof. F.P.T. Baaijens where he studied distributive mixing processes. After a six month leave at Océ Technologies he joined the Materials Technology group as an Assistant Professor.

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