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Rheological Properties of Drilling Fluids: Use of Dimensionless Shear Rates in Herschel-Bulkley and Power-law Models

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Abstract:
An approach of Nelson and Ewoldt [1] to create a viscosity model of the Herschel-Bulkley type in order to use only parameters with the potential of containing fluid information has been extended to be applied to drilling fluids using current industry standard procedures. The commonly used Herschel-Bulkley consistency parameter *k* is found inadequate in describing fluid properties properly as it has a unit dependent on *n*. Hence, the model is not optimum for digitalisation. The Herschel-Bulkley model is re-written and base its parameters directly on the yield stress and the additional or surplus shear stress at a pre-determined shear rate relevant for the flow situation to be considered. This approach is also applicable for Power-Law models.

Key words:
Herschel Bulkley model, power-law model, viscosity parameters, drilling fluids, Nelson and Ewoldt approach

1 INTRODUCTION

The current practice, independent on type of industry, is to use digitalised models for industrial processes whenever possible. Hence, different drilling processes can be controlled by use of simple computer applications or, in many cases, also mobile phone apps. To be applicable, these models must be reasonably accurate. This argument is also valid for drilling fluid viscosity measurements. Current drilling practice rely on standards like API [2] or ISO [3, 4]. Even though the number of measurement points may be limited, these standards base their viscosity models on measurements conducted at a wide range of shear rates. Earlier, the viscosity models were based on viscosity measurements at shear rates of 511 and 1022 1/s to create their viscosity data. These shear rates are far too large to represent practical drilling operations. It is recommended in the current standards to use a least square fit of all shear stress measurements, using their affiliated shear rates, to increase the accuracy of the viscosity models. However, with the exception of the flow around the Bottom Hole Assembly (BHA), shear rates in excess of 250 1/s are seldom experienced in the field [5, 6]. Therefore, an improved model accuracy will be obtained if the least square fit is conducted only for the relevant shear rates of the drilling fluid flow situation.

Several models are used to describe complex fluids like drilling fluids. The range of these models include simple two-parameter models like the Bingham model to complex models trying to encapsulate structure build-up and disruption like the Quemada model [7, 8]. Also viscoelastic properties can be important [5, 9, 10]. The simplest model that describes the flow curve with reasonable accuracy seems to be the Herschel-Bulkley model, named after Herschel and Bulkley [11], who described how such a flow curve should behave. In the Herschel-Bulkley model the shear stress is related to a yield stress τ_y, a consistency factor *k*, and the shear rate $\dot{\gamma}$ by the use of Equation 1.

$$\tau = \tau_y + k\dot{\gamma}^n \tag{1}$$

The yield stress is a property arising from the composition of the drilling fluid. This value will change dependent on several parameters; for example the number of particles of a certain size in the fluid. The unit of consistency factor *k* is dependent on the curvature exponent *n*, thus, $k = k(n)$. The consequence of this is that the parameter cannot be determined directly from the fluid measurements and must be identified through algebraic operations and that it cannot contain information

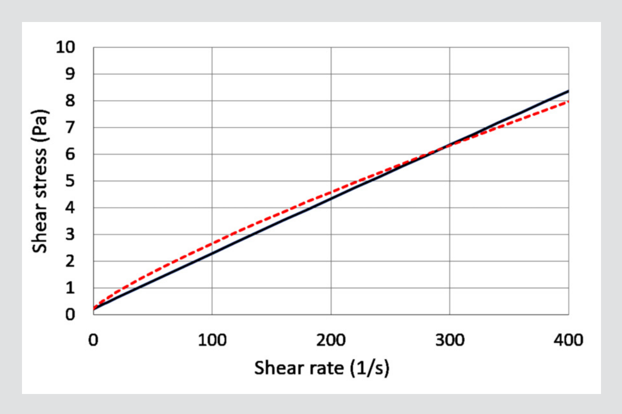

Figure 1: The viscosity flow curves of a fluid presented with different k and n with yield stress is 0.2 Pa (red curve: k = 0.0548 Pas^n and n = 0.8269, blue curve: k = 0.0229 Pas^n and n = 0.9806).

Least square fit shear rate range (1/s)	Yield stress, τ_y (Pa)	Consistency, k (Pa·s^n)	n (dimensionless)
0 - 1000	0.2	0.0548	0.8269
0 - 300	0.2	0.0229	0.9806

Table 1: Herschel-Bulkley parameters used in Figure 1.

about physical dependencies for the fluid. An example of relatively similar flow curves for different combinations of *k* and *n* is shown in Figure 1. Actually, both curves in Figure 1 are adopted from two approximations presented by Ytrehus et al. [12] for a field applied drilling fluid. The viscosity measurements were conducted on an Anton Paar MCR102 Rheometer. Both curves represent least square fit of rheometer data after the determination of the yield stress. The curve with *n* = 0.8269 are constructed on the basis of using all measurement values up to a shear rate of 1000 1/s, and the other by using only the measurement values up to a shear rate of 300 1/s to cover the laboratory experiment shear rates. The viscosity model of the latter approximation was found to reproduce experimental pressure loss data more closely when the annular pressure loss model by Founargiotakis et al. [13] for Herschel-Bulkley fluids was used.

Both approximations for the flow curve shown in Figure 1 represents the measurements relatively well even though the pair of parameters n and k are significantly different for these two curves. These parameters are tabulated in Table 1. It is shown that the numerical values of the consistency parameter developed by fitting measurements values from the shear rate range up to 300 1/s is less than half of the value obtained if all the measurement values up to 1000 1/s were used in the fit. This is an example of the fact that *k* cannot be used alone as a fluid property parameter. Its numerical value will always be dependent on the index *n*. Hence, the meaning of tabulating the parameter *k* for other perspectives than reproducing numerical calculations should be questioned.

Nelson and Ewoldt [1] presented a modified Herschel-Bulkley model with the scope of overcoming the limitations appearing when using the *k* and *n* approach. This model will be explained in the next section. Still, this model is not practical for describing drilling fluids in accordance with the API or ISO specifications. Therefore, based on this model a set of parameters is developed to modify the Nelson and Ewoldt parameters to be applicable to drilling fluid engineering. These parameters should be applicable to add other types of information like effects of vibration on the drilling fluids, which so far not have been properly modelled. The effects of vibration on drilling fluid flow curves have been described, but not the effect on other parameters than the yield stress [14].

2 USE OF NELSON AND EWOLDT'S PARAMETERS IN THE HERSCHEL-BULKLEY MODEL

Nelson and Ewoldt [1] found that by applying the consistency parameter *k*, which was dependent on *n*, it was not possible to compare different materials. Hence, parameters are needed that are more universal than the traditional Herschel-Bulkley parameters. Nelson and Ewoldt [1] developed an alternative parameter to the consistency parameter *k*. Their first step was to determine the yield stress from the viscosity flow curve. The yield stress is a fluid structural parameter. Then, they selected the shear stress and affiliated shear rate at which the shear stress is twice the yield stress, named the critical shear rate.

$$2\tau_y = \tau_y + k\dot{\gamma}_c^n \tag{2}$$

By introduction of this relation into the original Herschel-Bulkley equation (Equation 1) they obtained a Herschel Bulkley equation where all parameters are independent.

$$\tau = \tau_y\left[1+\left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^n\right] \tag{3}$$

Equation 3 is a presentation of a Herschel-Bulkley fluid in a form where all the parameters τ_y, $\dot{\gamma}_c$, and *n* can be treated separately. Furthermore, the shear rate enters a dimensionless form in the equation. When measured properly with small enough increments, these parameters can all be determined directly from measurements. Measurements of the field applied drilling fluid shown in Figure 1 show that this drilling fluid had a very low yield stress. The yield stress of some drilling fluids are too low to be determined by current oil well drilling standard procedures. These drilling fluids can be sufficiently well described using the power-law model that do not exhibit any yield stresses. Furthermore, determination of particular shear stresses from use of conventional viscometers used in accordance with API/ISO specifications is not practical as these standards specify measurements at a very limited number of shear rates. Also, the accuracy of these conventional VG meter measurements at low shear rates can be questioned. Therefore, it is not practical to use the parameters suggested by Nelson and Ewoldt [1] for drilling fluids. However, it is straightforward to expand their parameters to be used in drilling fluids.

3 EXTENSION OF NELSON AND EWOLDT'S PARAMETERS TO BE USED IN HERSCHEL-BULKLEY MODELS FOR DRILLING FLUIDS

By selecting a relevant shear rate for the flow that shall be described using the Herschel-Bulkley model, it is straightforward to expand Nelson and Ewoldt's approach and use parameters that can more easily be used in digitalised models. The first item is to approximate the yield stress from the viscosity flow curve. The next item is to determine a surplus stress τ_s at a specified shear rate. As an example, a shear rate of 170.3 1/s will be selected in the next section. This shear rate is a relevant for many drilling operations. This shear rate is equivalent to that obtained at 100 RPM on most conventional viscometers currently described by API [2] or ISO [3, 4]. At the same time it is not too far outside the applicable shear rate range for drilling fluid circulation.

$$\tau = \tau_y + \tau_s \left(\frac{\dot{\gamma}}{\dot{\gamma}_s} \right)^n \tag{4}$$

where

$$\tau_s = \tau - \tau_y \quad at \quad \dot{\gamma} = \dot{\gamma}_s \tag{5}$$

For a power-law fluid, Equations 4 and 5 are still valid by setting $\tau_y = 0$. If this equation is reversed such that the shear stress is set and the shear rate is measured, the Nelson and Ewoldt [1] model is obtained by setting $\tau_s = \tau_y$ and the corresponding $\dot{\gamma}_c = \dot{\gamma}_s$. The curvature exponent can be found for example by using Equation 6. In this case $\dot{\gamma}_x$ is a selected shear rate where τ_x is measured. This shear rate should in principle be within the relevant shear rate range for the flow problem to be evaluated.

$$n = ln\left(\frac{\tau_x - \tau_y}{\tau_s} \right) \Big/ ln\left(\frac{\dot{\gamma}_x}{\dot{\gamma}_s} \right) \tag{6}$$

The curvature exponent n as presented in Equation 6 will change if different shear rates are used to determine either n or τ_s. Therefore, the shear rates used to calculate n and τ_s must be specified. In principle it is now possible to tabulate information or pressure and temperature to n and τ_s measured at these specified shear rates. In general, it will not be possible to correlate data measured at a particular shear rate directly to n and τ_s measured at other shear rates. However, this approach will allow the industry to compare fluids planned for use in different well sections as these section have typical maximum shear rates during drilling fluid circulation.

4 APPLICATION EXAMPLE

In the following example the viscosity of drilling fluid laboratory samples made for field application was evaluated. For the ease of understanding the example is made from direct application of simple measurements with field drilling fluids. Note that improved results may be obtained if results are fitted to the model prior to the analysis in the example as is described in the following chapter. Measurement data were collected at the API [2] and ISO [3, 4] specified viscometer rotation rates. These rotation rates and the corresponding shear rates are tabulated in Table 2. The first step is to approximate the yield stress from the flow curve. In the current examples the yield stress is approximated following Zamora and Power [15] as:

$$\tau_y = 2\tau_3 - \tau_6 \tag{7}$$

RPM	3	6	30	60	100	200	300	600
1/s	5.11	10.22	51.1	102.2	170.3	340.7	511	1022

Table 2: Conversion of VG meter RPM to shear rates (1/s).

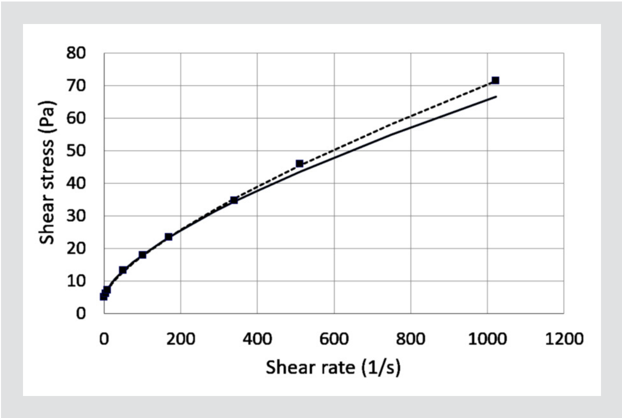

Figure 2: Comparison of measurement values with model predictions for an oil based drilling fluid measured at 20°C (τ_y = 4.6 Pa, τ_s = 15.33 Pa, and $\dot{\gamma}_s$ = 170.3 1/s). Solid line represents results calculating $n = n_{ls}$ = 0.695 at $\dot{\gamma}$ = 50.11 1/s and the dotted line represents the results calculating $n = n_{hs}$ = 0.796 at $\dot{\gamma}$ = 1022 1/s.

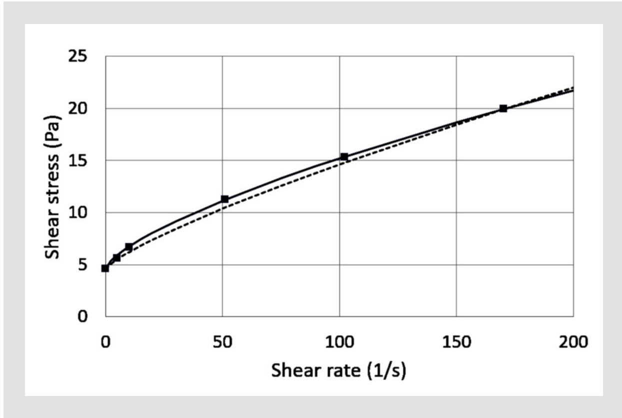

Figure 3: Comparison of measurement values with model predictions for an oil based drilling fluid measured at 20°C (τ_y = 4.6 Pa, τ_s = 15.33 Pa, and $\dot{\gamma}_s$ = 170.3 1/s). Solid line represents results calculating $n = n_{ls}$ = 0.695 at $\dot{\gamma}$ = 50.11 1/s and the dotted line represents the results calculating $n = n_{hs}$ = 0.796 at $\dot{\gamma}$ = 1022 1/s. The figure is an enlargement of the low shear rate regime of Figure 2.

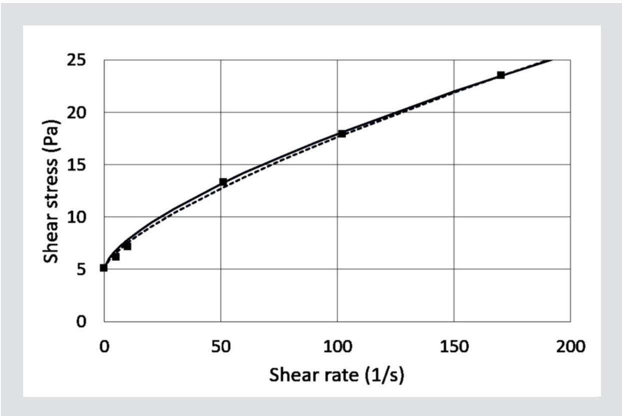

Figure 4: Comparison of measurement values with model predictions for a water based drilling fluid measured at 20°C (τ_y = 5.11 Pa, τ_s = 18.4 Pa, and $\dot{\gamma}_s$ = 170.3 1/s). Solid line represents results calculating $n = n_{ls}$ = 0.674 at $\dot{\gamma}$ = 50.11 1/s and the dotted line represents the results calculating $n = n_{hs}$ = 0.717 at $\dot{\gamma}$ = 1022 1/s.

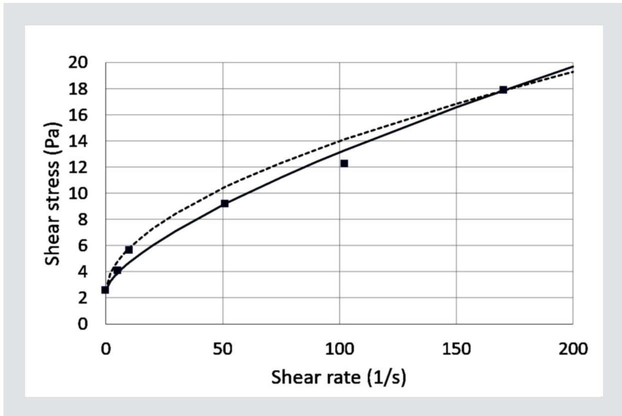

Figure 5. Comparison of measurement values with model predictions for a water based drilling fluid measured at 50°C (τ_y = 2.56 Pa, τ_s = 15.33 Pa, and $\dot{\gamma}_s$ = 170.3 1/s). Solid line represents results calculating $n = n_{ls}$ = 0.695 at $\dot{\gamma}$ = 50.11 1/s and the dotted line represents the results calculating $n = n_{hs}$ = 0.547 at $\dot{\gamma}$ = 1022 1/s.

In the examples the numerical subscripts refer to the particular rpm of the conventional viscometers described in API [2] and ISO [3, 4] procedures, albeit the fact that only metric units are used in the calculations. The next step is to measure the shear stress at 170.3 1/s. This value was chosen as it represents a typical upper limit for a lot of practical annular flow cases [5, 6]. Then the yield stress was subtracted to give the surplus shear stress τ_s.

$$\tau_s = \tau_{100} - \tau_y \tag{8}$$

This parameter does no longer contain any dimension dependent on the curvature exponent *n*. Finally, the curvature exponent *n* is determined. In the present cases two possibilities were chosen. One should have the optimum accuracy at the shear rates less than $\dot{\gamma} = \dot{\gamma}_s$ = 170.3 1/s and the other one at higher shear rates. The curvature exponents for these two cases are defined as n_{ls} for the low shear exponent and n_{hs} for the high shear exponent. In the current example the measurement at 30 rpm (51.1 1/s) is used to produce n_{ls} and the measurement at 600 rpm (1022 1/s) is used to create n_{ls}. Hence, these two values are calculated as:

$$n_{ls} = ln\left(\frac{\tau_{30} - \tau_y}{\tau_s}\right) \Big/ ln\left(\frac{\dot{\gamma}_{30}}{\dot{\gamma}_s}\right)$$

$$n_{hs} = ln\left(\frac{\tau_{600} - \tau_y}{\tau_s}\right) \Big/ ln\left(\frac{\dot{\gamma}_{600}}{\dot{\gamma}_s}\right) \tag{9}$$

In Figure 2 it is shown the shear stress as function of shear rate for a comparison of measurement values

$\dot{\gamma}$	Data for Figures 2 and 3			Data for Figure 4			Data for Figure 5		
	Measured	n_{ls}	n_{hs}	Measured	n_{ls}	n_{hs}	Measured	n_{ls}	n_{hs}
5.11	5.62	5.94	5.54	6.13	6.84	6.60	4.09	3.90	4.80
10.22	6.64	6.77	6.22	7.15	7.88	7.56	5.62	4.73	8.84
51.1	11.2	***11.2***	10.5	13.3	***13.3***	12.9	9.20	***9.20***	10.5
102.2	15.3	15.4	14.8	17.9	18.2	17.9	12.3	13.3	14.1
170.3	19.9	***19.9***	***19.9***	23.5	***23.5***	***23.5***	17.9	***17.9***	***17.9***
340.7	30.7	29.4	31.2	34.7	34.5	35.3	23.0	27.4	25.0
511	40.9	37.5	41.4	46.0	43.7	45.5	28.6	35.4	30.5
1022	68.5	57.8	***68.5***	71.5	66.6	***71.5***	43.4	55.8	***43.4***
τ_y		4.60	4.60		5.11	5.11		2.56	2.56
τ_s		15.3	15.3		18.4	18.4		15.3	15.3
n_{ls}		0.695			0.674			0.695	
n_{hs}			0.796			0.717			0.547

Table 3: Measured and modelled data used in Figures 2–5. Numbers in bold face italics are exact values used to calculate τ_s and n. Unit for all data is Pa.

with model predictions for an oil based drilling fluid. The yield stress was τ_Y = 4.6 Pa, the surplus shear stress τ_s = 15.33Pa at the pre-determined shear rate $\dot{\gamma}_s$ = 170.3 1/s. The solid line represents results that are more accurate at lower shear rates using $n = n_{ls}$ = 0.695. The dotted line the shows the results that are more accurate at higher shear rates with the exponent $n = n_{hs}$ = 0.796. To compare these results at the lower shear rates it is practical to evaluate Figure 3 which represents a low shear rate magnification of Figure 2.

It is shown in Figure 3 that with the selected shear rates, all models have identical yield stress values and equal values at the shear rate of 170.3 1/s. At the shear rate of 51.1 1/s the curve with $n = n_{ls}$ has an identical value as the experiments. This specific measurement was selected to determine the value of the exponent *n*. Similarly, it is observed in Figure 2 that with $n = n_{hs}$ an identical value as the experiments is obtained at the shear rate 1022 1/s. While working with drilling fluids it is easy to get the impression that a Herschel-Bulkley modelled flow curve develop lower values if the model is based on the lower shear rate values like what is shown in Figure 3. This is, however, not generally true. For the curve shown in Figure 3, the predicted shear rate values for the curve with $n = n_{hs}$ give a lower shear stress value at the lower shear rates with $n = n_{ls}$. The flow curves shown in Figure 4 and Figure 5 illustrate a different behavior. Because the values at the shear rate at 170.3 1/s is identical (see Table 3), the curve with the lowest *n* value will show the lowest shear stress at lower shear rates and highest at higher shear rates. However, the curves may have its pronounced curvature at different shear rates. First of all, as was shown already in Figure 1, two curves with different shear stresses may give approximately the same results. This is also shown in Figure 4 for a set of measurements on a water based drilling fluid at 20 °C. When the measurements of this water based drilling fluid were conducted at a temperature of 50 °C, the situation is different. First the yield stress is reduced to the half. The surplus stress at 170.3 1/s is also reduced. However, the shear rate range with the highest curvature has been altered. The most accurate results at lower shear rates are now found using $n = n_{ls}$ = 0.695, which is larger than $n = n_{hs}$ = 0.547 that predicts more accurate results at higher shear rates. This is illustrated in Figure 5.

5 COMMENTS TO THE RESULTS

Traditionally, the low shear rate experiments using manual equipment may have large uncertainties. That will question the measured yield stress. However, by use of more accurate instruments, the determination of the yield stress can be improved. All current measurements using *k* and *n* may be useful in the proposed model if the *k* value is transferred to a surplus shear stress value. It is also a benefit if the models are optimised by curve fitting within the relevant shear rate range. Such curve fit models will normally reduce some of the uncertainties introduced by using the standard measurement procedures. The surplus shear stress can be calculated as:

$$\tau_s = k\dot{\gamma}_s^n \tag{10}$$

If Equation 10 is inserted into Equation 4, the original Herschel-Bulkley equation is resumed. The current practice to describe the drilling fluids with *k* and *n* hinders optimum digitalisation process within the drilling industry. By changing to evaluate the yield stress, the surplus stress and the curvature exponent at a relevant shear rate, the parameters can easier be related to temperature, pressure, chemical content, and particle addition. Hence, this approach will give the field engineer a better understanding of the function of the viscosity model.

6 CONCLUSIONS

A viscosity model of the Herschel-Bulkley type where the shear rate is made dimensionless by selecting a characteristic shear rate for the flow has been presented. An example is presented where this approach is used on drilling fluids. The Herschel-Bulkley parameters may then have the potential of containing fluid information and be compared with other fluids. The Herschel-Bulkley consistency parameter *k* is found inadequate in describing properties in a simple way as it has a unit dependent on *n*. Hence, the model is not optimum for digitalisation. The Herschel-Bulkley model could be re-written as $\tau = \tau_y + \tau_s (\dot{\gamma}/\dot{\gamma}_s)^n$ where τ_y is the yield stress and $\tau_s = \tau - \tau_y$ at the pre-determined $\dot{\gamma} = \dot{\gamma}_s$. The proposed method is equally good for the description of power-law models; simply by setting the yield stress equal to zero.

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